Compendium Euclidis Curiosi:

OR,

GEOMETRICAL

OPERATIONS.

Shewing how

With one single opening of the COMPASSES and a straight RULER all the PROPOSITIONS of *EUCLID*'s first Five Books are performed.

Translated out of *Dutch* into *English*,

By *Joseph Moxon*, Hydrographer to the Kings most Excellent Majesty.

LONDON:

Printed by *J. C.* for *Joseph Moxon*, and sold at his shop on *Ludgate-hill* at the signe of *Atlas*; and by *James Moxon*, neer *Charing-cross* in the *Strand*, right against King *Harry* the Eighth's Inne. 1677.

TO

The Right Worshipful

Sir CHARLES SCARBOROUGH, Kt.

SIR,

YOur great proficiencie in the excellent Science of *Geometry*, together with the long experience I have had of your Civi-

 lity

lity and good nature, hath emboldened me to preſent you with this *Engliſhed Dutch Rarity.* And though it be ſmall in it ſelf, yet your acceptance will not onely make it great, but alſo crown the deſires of

Sir,

Your moſt humble Servant,

JOSEPH MOXON.

THE AUTHOR'S PREFACE TO THE READER.

It is very well known that the Judgements of men are changeable, and that at one time they will reject, through Ignorance, what at another time their Reason will cry up; and this many times proves prejudicial. Thus it hath happened to me: For having read in Peter Ramus his Dutch Geometry, printed at Amsterdam 1622. fol. 44. how that one John Baptista hath set forth a Book wherein with one single given opening of the Compasses, he answers all Euclid's Propositions and Operations.

But

But that Book I could never yet get sight of. Therefore hath my curiosity prompted me to know the same; but when my thoughts fell upon the Two and twentieth Proposition of Euclid's *first Book, how of three given right Lines, equal to three given right Lines, to make a Triangle; but two of them (after what manner soever they are taken) ought to be greater than the other; for as much as of every Triangle two sides (after what manner soever they be taken) are greater than the other side. I confess I thought it was not to be done; and upon inquiry I found many skilful men of the same Opinion, also that the aforesaid Book never came forth; and that there were several* J. Baptista's *that have written*, &c. *whereof* Marinus Bettinus *in his* Ærarium Philosophiæ Mathematicæ, *printed at* Bononia 1648, *makes mention: And if the aforesaid Book should have come forth, the foresaid Author would doubtless have taken notice of it. But he hath performed several Propositions of* Euclid's *first Book, with once opening of the Compasses, as* 1, 9, 10, 11, 12, *and* 31 *Propositions. So also hath* Peter Wils, *in his* Geometrical *Works, printed at* Amsterdam 1654, *fol.* 62. *shewn how in a Circle to finde all inscribed equal-sided Figures sides, from a figure of three Angles to one of twelve Angles: by keeping the Compasses at the same width the Circle was made off, (which* Daniel Schwenter *also in his* Geometry *hath printed at* Nurenburg 1641, *fol.* 208.) *This Author does indeed shew the length of the Line: For example,*

he

he shews in a Pentagon, as in the Four and twentieth Proposition of this Treatise D E *is done, but not on its place, as* G D, *&c. So hath also* Christopher Nottnagel *in his* Manuel Architecturæ Militaris, *printed at* Wittenberg 1659, *fol.* 177, *shewn how a Regular Fortification, without Arithmetick and by one single opening of the Compasses, may be described on Paper; which is the Two and thirtieth Proposition of this Treatise.*

These are all the Authors, ever came to my knowledge, that have written on this Subject; but they all use the Compasses opened to such a width, as one of the Lines (except Bettinus *in some of his:) Neither do they scruple the using of a Line, though there be great difference in the Operation between a pair of Compasses and a Line, and a pair of Compasses alone; as may be seen in a Treatise named* Euclides Danicus, *set forth by* George Mohr, *and printed at* Amsterdam 1672.

Upon these considerations have I found an impulse upon me to expose this small Work to publick view, (especially since several have written on this Subject; and though all have aimed at it, yet none could hit the White, as aforesaid) How onely with one given width of the Compasses and a straight Ruler (which I will use at my own pleasure to prolong a Line with) to demonstrate the Propositions of Euclid; *as you may see in this small Treatise.*

And though these Propositions may be performed other ways, yet have I here set down only the most singular, that may be easiliest understood.

To the Reader.

I would have added more Operations (as well how to perform the ſame with one given Line, and one given opening of the Compaſſes; as alſo with a given Semi-Circle, or, &c.) as alſo how to deſcribe Sundyals of what ſort ſoever : but conſidering that all Flat or Plain Operati ons may be reduced from theſe, theſe ſhall ſuffice.

And if I finde this prove acceptable to you, then with good reaſon ſhall I publiſh ſomething elſe of another nature.

In Amſterdam,
March 1. 1675.

Compen-

N: B. the great Fault in this Book is that the Author or Translator has neither prefixt nor Annext the Scheme or Schemes whereby all his Propositions are wrought. but I have since got it & is here loose

Compendium Euclidis Curiosi:

OR,

GEOMETRICAL OPERATIONS:

Performing all the PROPOSITIONS of the first Five Books of *EUCLID*, with one single opening of the COMPASSES and a straight RULER.

PROPOSITION I.

To divide the given Line A B *into two equal parts.*

OPERATION.

FRom the ends of the Line, as at A B, set off the width of your Compasses so oft till C D become shorter than your width. Then on the Points C D describe cross Arches, as E F. Draw a straight Line, as E F, through the inter-section of these Arches, and this straight Line shall divide your given Line A B into two equal parts.

Otherwiſe:

From the ends of the Line, ſet off from A to G, and from B to H, the width of your Compaſſes, and on G and H deſcribe the Arches A I and B K, the one above the Line A B, the other below the Line A B. Then ſet off in theſe two Arches the width of your Compaſſes from A to I, and from B to K, and through theſe two Points of ſetting off, draw the ſtraight Line I K, which ſhall divide the given Line A B in M into two equal parts.

PROPOSITION II.

Upon the given Line A B *to erect a Perpendicular at the Point* C.

OPERATION.

On the Point C deſcribe an Arch; in this Arch ſet off your width twice, as from D to E, and from E to F. On the Points E F deſcribe two Arches, as at G, and draw a ſtraight Line through the interſection of theſe Arches to the Point C; which ſtraight Line is the deſired Perpendicular.

PROPOSITION III.

Upon the given Line A B *to deſcribe an equilateral Triangle.*

OPERATION.

1. On the ends A and B ſet off the width from A to D, and from B to E; and on the Points D E deſcribe

ſcribe croſs Arches, as F G: Then draw ſtraight Lines from A through F, and from B through G; and the meeting of thoſe two ſtraight Lines, as at C, ſhall make the equilateral Triangle A B C.

2. But if your width A D and B E be longer than the Line A B, deſcribe on the Points A B the croſs Arches F G, and draw ſtraight Lines from A to F, and from B to G, which ſhall cut each other in the Point C, and make the equilateral Triangle A B C.

3. Otherwiſe: On the Line A B (or its prolongation, if A B be ſhorter than A D) ſet off at A the width A D. On thoſe two Points A and D, deſcribe the Croſs-arches E, through which a ſtraight Line drawn from A to the Perpendicular G H (erected on the middle of A B, as by Prop. 1,2.) as at C, and another ſtraight Line from C to B, ſhall make A B C the equilateral Triangle.

PROPOSITION IV.

From a given Point to draw a Perpendicular through a given ſtraight Line.

OPERATION.

The given Point is A, the given ſtraight Line is B C. Place one foot of your Compaſſes on the Point A, and with the other deſcribe the Arch C E. On theſe Points D E deſcribe the Croſs-arches G, and draw a ſtraight Line from A to G, which ſhall be a Perpendicular to B G.

2. But if from A the extent of your Compaſſes will not cut the right Line B G, then erect at your

 plea-

pleaſure two perpendiculars, as H K, and I L (as by the ſecond Operation) and on the Points K and L, ſet off your width ſeveral times, till from A you may cut the ſtraight Line M N, and then work as before.

PROPOSITION V.

Through the given Point A *to draw a Line parallel to the given Line* B C.

OPERATION.

Place one foot of your Compaſſes at A, and with the other deſcribe the Arch G H, which will cut B C at G: from G ſet off your width as at F, and from F deſcribe a Croſs-arch to cut the Arch G H, as at E; then draw a ſtraight Line through the Points A and E, which ſhall be the Parallel-line required.

But if the width will not from A cut B C, then let fall the Perpendicular A D, from the Point A, upon B C, as by the former Operation; and on the Point A erect a Perpendicular to A D, as by Oper. 2. which laſt Perpendicular A E, ſhall be the Line parallel to B C.

PROPOSITION VI.

To joyn the right Line C D *to the right Line* A B, *at the end* B *in a right Line with* A B.

OPERATION.

1. By the laſt Operation draw the ſtraight Lines G K

G K parallel to C D, and D E parallel to C B ; then ſet off on the prolonged Lines G B and A B from the Point B, the width as B F and B H ; and from the point E draw a Line parallel to F H, which will cut the prolonged Line A B at P ; then is B P as G D joyned in a right Line with A B. If you will joyn C D at the end A, or A B to C D, the Operation is the ſame.

2. Obſerve : C D may be made that C D and B or C D and A ſtand in a right Line, thus : Draw the Lines C E from C parallel to A B, alſo B E from B parallel to C D ; and from C in the Lines C E and C A, ſet off the width C F and C H ; then draw the Lines D Q parallel to F H, and Q P parallel to C A ; ſo ſhall P A B be the required Line. Or draw the Line E R from E parallel to H F, then is R C as A B, joyned to D.

3. But if C D be equal to A B, then erect the Perpendicular B K on the Point B in the Line A B, by the ſecond Operation ; and on that Perpendicular ſet off the width twice from the Point B, as at B E and E F ; from the Point F draw the Line F L Perpendicular to B F : Then if you draw a ſtraight Line through A and E, and prolong it to the Line F L, as at G, and from G draw a ſtraight Line G H parallel to B F, which will cut the prolonged Line A B in H ; then is B H equal to B A. This manner of working may alſo be uſed if you would make A B twice the length from A : and if you would make A B thrice the length from A, then ſet off the width three times on B K, and draw the Line through A E, as aforeſaid.

4. Or

4. Or if you would joyn part of the given Line H Q (as A Q) to the other end of H Q, ſo that H D be equal to A Q, then is firſt erected the Perpendicular B K on the middle of A H, by the firſt and ſecond Operation, and wrought afterwards as you are taught by the third Section of this Operation.

PROPOSITION VII.

From a given right Line, as A B, *to cut off a ſhorter right Line, as* C D.

OPERATION.

Draw the Line G K through B parallel to C D, and the Line C E from C parallel to B D, as by the fifth Operation. Set off the width on the prolonged Lines G K and A B, from B to B F and B H; then draw from E the parallel to F H, which ſhall cut B A in P: Then ſhall P B be equal to C D, and A P the overplus. The other conſiderations may be eaſily underſtood by the foregoing Operations.

PROPOSITION VIII.

Upon the end of a given right Line, as A B, *to place another right Line, as* C D, *perpendicularly.*

OPERATION.

By the ſixth Operation joyn C D to A B, and the product will be A P; hereon at B erect a Perpendicular, as B G, at pleaſure, by the ſecond Operation: On

On this Perpendicular at B, ſet off the width, as B F, as alſo from B in the Line B P at E: From P draw the parallel to E F, to cut B G in M; ſo ſhall B M be equal to C D, and alſo perpendicular to A B.

PROPOSITION IX.

To divide a given Line, as A B, *into three equal parts: Or to cut off one third part.*

OPERATION.

On the ends A and B, draw the Perpendiculars A C and B D (by the ſecond Operation) or make the Angles equal (as by the ſecond manner of the firſt Operation.) On theſe Perpendiculars ſet off from A and B the width twice (or once leſs than the Line is to be divided into equal parts) as at G and H: Then draw the right Lines E H and G F, which ſhall cut A B in I and K: ſo ſhall A I be equal to I K, and K B equal to both, as was propoſed.

PROPOSITION X.

To two given right Lines, as A B *and* B C, *to finde a third proportional; that is, as* AB *to* BC, *ſo ſhall* B C *be to* D E.

OPERATION.

1. Joyn the Lines A B and B C, as by the ſixth Operation; and at A ſet off the Line A D equal to B C, making an Angle at pleaſure with A C: then from

from C draw a Line parallel to B D, which ſhall cut the prolonged Line A D in E ; ſo ſhall D E be the third proportional.

2. To divide the given Line A E in proportion as A B is to B C, Joyn A B and B C as by the ſixth Operation, and at A draw the Line A E to make an Angle with A C at pleaſure. Then draw the Line B D from B parallel to C E ; ſo ſhall A E be the part proportional.

3. Otherwiſe. On A B from A and E make Angles, as in the ſecond manner of the firſt Operation. Then draw the right Lines A G and E H, and ſet off on them A B and E C, (E C is equal to the foregoing B C) as by the Sixth Operation. Then draw the right Line B C, which ſhall cut A E in D as is required.

PROPOSITION XI.

To three given right Lines, A B, B C, *and* C D, *to finde a fourth Line* E F *in proportion as* A B *to* B C, *and as* C D *to* E F.

OPERATION.

Having joyned the Lines A B and B C, and made an Angle at pleaſure from A with the Line A E, equal to C D, Then draw from C a parallel to B E, as by the fifth Operation, which ſhall cut the prolonged Line A E in F. So ſhall E F be the fourth proportional.

PRO-

Proposition XII.

From two right Lines A *and* B, *to finde the mean proportional* B M.

OPERATION.

1. Joyn the Lines A B and C D, (as by the ſixth Operation) and it will make the Line A P. Then make the Angle P A L at pleaſure from A to A P; and ſet off the width A H to F twice, and once in the prolonged Line B A from A to T; and on the Point T deſcribe the Semi-circle A K V: Then draw the Lines B G and G R parallel to P F and F V, from B and G by the fifth Operation; as alſo the perpendicular B S (by the ſecond Operation) on R B from B, and R W from R, which ſhall cut the Semi-circle in K; from hence a right Line drawn through A, and prolonged till it cut B S in M, gives B M for the mean proportional required.

2. Or, on the Center E of the joyned Lines A B and C D deſcribe with the width the Semi-circle, and make your Angle P E Q at pleaſure, and ſet off the width on E Q from E to F: Then draw the lines B H and H K parallel to P F and F G; and erect on A B at B and K the perpendiculars B N and K R, which perpendicular K R ſhall cut the Semi-circle in L, and a right line drawn from E through L, and prolonged to the line B N as at M, gives B M for the mean proportional required.

Proposition XIII.

To change the given right Angle A B *into a Square* C D.

OPERATION.

By the former Operation you may finde the required ſide B M.

Proposition XIV.

Of three given right Lines A F, B F, *and* A B, *whereof two are longer than the third, it is required to make a right lined Triangle* A F B, *upon one of the given right Lines* A B.

OPERATION.

1. Joyn to the half of A B the third Proportional by Operat. VI, X. That is, as twice A B is to D F, ſo is A F to the Product, and the remainder is B G: this joyned and deducted from B F (by Operat. VI, and VII.) and the mean proportional F G, found by Operat. XII, of the longeſt and ſhorteſt piece, that is, B F+B G, and B F—B G: Then draw the right Lines from A and B to F, ſo is the Triangle A F B made.

Obſerve. This Rule may be fitted to all Triangles that may be deſcribed on the longeſt ſide: but in the Sequel you will finde by the working that you cannot elect what ſide you will to deſcribe your Triangle upon. This granted, you will have on the ſide

ſide (for Example A B) of an equal leg'd Triangle that comprehends the equal legs, no more to do to finde A G, but to divide A B in the middle at G, by Operat. I. Then work as before.

2. If it be a right angled Triangle, you may joyn two of the ſides; the third will give it ſelf. For Example. To finde the Hypothenuſe A F. (by Operat. VIII) Place one of the ſhorteſt ſides F G perpendicularly upon the end of the other A G: Then draw A F, ſo ſhall A G F be the required Triangle: but if the Hypothenuſe A F and one of the ſhorteſt ſides (ſuppoſe A G) are known, you will finde the third ſide F G, which is the mean proportional between A F+ A G and A F— A G.

3. In obtuſe angled Triangles is this difference from acute angled Triangles: from the obtuſe Angle, and from the ſum ſubſtracted from the third Proportional.

PROPOSITION XV.

From the given Point C, *to draw a ſtraight Line to touch the given Arch* A B.

OPERATION.

Having joyned the fourth Proportional E B (which is as AC to A B and A H, ſo is B C to E by Operat. XI, VI.) to B C, erect a Perpendicular on E B at E, which will cut the Arch at F. Or if you have no Arch drawn, then finde the mean Proportional E F between A E and E C (by Operat. XII) and draw a right Line from C to F, which is the right Line required.

2. But if you have onely the Diametral line and the Point C given, (be it above, or below, or on either ſide the Diametral-line) draw a right line from C through A the middle of B H, as C G, and ſet off from A in the line A G and A H the width A M, equal to A L: Then draw the lines B N and H O from B and H parallel to L M, (by Operat. V.) ſo ſhall O N be equal to H B. Work the reſt as you were taught before.

PROPOSITION XVI.

To divide the given right lined Angle B A C *into two equal Angles.*

OPERATION.

Set off on A B and A C the width A D and A E, and make on the Points D E the croſs Arches F, through which from A draw a right line which cuts the Angle B A C into equal Angles. If the Angle B A C be very obtuſe, deſcribe with the width the Arch D E on A, and ſet off the width from D and E in the Arch, as at G, H; then on G H make the croſs Arches K, and draw a ſtraight Line through them and A, which ſhall divide the Angle into two equal parts.

PROPOSITION XVII.

On the given Line E F *at* E, *to make an Angle* H E F *(and* H E M) *equal to a right lined Angle* C A B *(and* C A L.)

OPERA-

OPERATION.

In A B at A ſet off the width as A D: hereon at D erect a Perpendicular D K, as by Operat. II. which will cut A C at I. Then ſet off the width on E F from E to G; and on G erect the Perpendicular G H equal to D I, (as by Operat. VIII.) and draw through H and E the right Line E H, ſo ſhall the Angles H E F and H E M be equal to the given Angles.

Note, If the Angle B A C be very neer a right Angle, the Perpendicular D I will be very long: Therefore ſet off the width alſo on A C from A to A P, and then on E G (equal to A P) make the Triangle E G N equal to A D P (by Operat. XIIII.) So have you the required Angle.

PROPOSITION XVIII.

In a given Circle, to make a right Line as A E *equal to a given right Line* C D, *not longer than the Diametral Line* A B.

OPERATION.

By Operat. XIIII. make on A B a right-angled Triangle, whoſe one ſhort ſide of the Triangle is A E equal to C D.

PROPOSITION XIX.

To make a Triangle, as P H O, *in a given Circle whose Semi-diameter is* H G, *that ſhall be alike in form with the given Triangle* D E F.

OPERATION.

Having by Operat. II. drawn the Perpendicular KHL on HG at H, and hereon at H the Angles MHL and NHK (by Operat. XVII.) made equal to the Angles E D F and E F D; HN and HM ſhall cut the Circle at O and P. But if no Circle were drawn, then let fall the Perpendicular from G on NH, and MH, as G R and G Q, (by Operat. IIII.) and then P R equal to H R joyned to R H, and Q O equal to Q H to H Q (by Operat. VI.) and laſtly, the right Lines drawn through P and Q ſhall form the Triangle P H O required.

2. Or upon the given Line A B to deſcribe a Triangle alike in form to a given Triangle D E F.

OPERATION.

Make the Angles B A T and A B V on A B at A and B, equal to the Angles F D E and D F E; (by Operat. VII.) then ſhall A T and B V, or their Prolongeds, if need requires, cut each other in C; ſo ſhall the Triangle A C B be the Triangle required.

PROPO-

PROPOSITION XX.

To make the Triangle XVY *about a given Circle, whose Semi-diameter is* M L, *that shall be alike in form with the given Triangle* G I H.

OPERATION.

On M L at M draw the Perpendicular L N and L O by Operat. II. and make by Operat. XVII. on M L from M the Angles L M Q and L M R, equal to the outer Angles K G I and P H I, so shall M Q and M R cut the Circle in T S.

Or, If no Circle be described, set off the width M B and M A equal to M C from C in M L, M Q, and M R: and draw from L by Operat. V. the Lines T L and S L, parallel to A B and B C, which shall cut M Q and M R in T S; hereon draw Perpendiculars to cut each other in V X Y, so shall the Triangle X V Y be alike in form with the Triangle G I H.

PROPOSITION XXI.

To describe a Circle in the Triangle A B C.

OPERATION.

Divide two of the Angles into two equal parts by Operat. X V I, which shall cut each other in the middle point D, from whence let fall the Perpendicular upon one of the sides, as A C, by Operat IIII. so shall the Perpendicular D E be the Semi-diameter

of

of the deſired Circle. Now ſhould the Circle be deſcribed with the given width of the Compaſſes; but becauſe this Line D E may be longer or ſhorter than the width, therefore it is unpoſſible to deſcribe a Circle bigger or leſſer than the given width on the middle point of the ſame right Plane: But we may finde almoſt infinite number of Points ſeveral ways, that will fall in the circumference, by which a Circle may be made: but ſuch a Circle we need not make, becauſe we may by the other ſeveral Operations finde ſuch Points as we may have occaſion to uſe.

PROPOSITION XXII.

On a given right Line I H, *to deſcribe a Square.*

OPERATION.

By Operat. VIII. erect on H the Perpendicular H G equal to I H; upon I H then draw Lines from G and I parallel to H I and H G, by Operat. V. to cut each other at F; ſo ſhall F G H I be the required Square.

PROPOSITION XXIII.

In a Circle, whoſe Semi-diameter is A B, *to deſcribe a Square.*

OPERATION.

Having drawn B D and B E equal to A B perpendicular from B, by Operat. VIII. then draw A D, C D, C E, and E A, the required Square. PRO-

Proposition XXIV.

To describe a Square about a Circle, whose Semi-diameter is A B.

OPERATION.

First describe a Square within a Circle, as by the last Operation; then draw from D, E, A, and C, lines parallel to A C and D E, by Operat. V. so shall F G H I be the required Square.

Proposition XXV.

In a given Circle, whose Semi-diameter is A B, *to describe an equilateral Pentagon.*

OPERATION.

On A B at A, erect the perpendicular A D equal to A B, by Operat. VIII. and draw a line from D to E, the middle of A B, as D E; joyn this to E at E B, by Operat. VI. so shall D F be the side of the Pentagon. If you now make the Triangle G A D on A D, whose sides A G shall be equal to A D, and G D equal to D F, by Operat. XIIII. then shall the ends of this Triangles side G D fall in the circumference of the Circle. Make the other Triangle D A M, which is equal to, and alike formed with the Triangle D A G, by Operat. XVII. making the Angle D A M equal to the Angle D A G; then on A D and A M from A, set off the width A N equal to

A O, and draw the line D M from D parallel to N O, by Operat. V. so shall D M be equal to G D: and thus you may work for finding the other sides.

Proposition XXVI.

To describe a Pentagon about a Circle, whose Semi-diameter is A B.

Operation.

Describe a Pentagon by the former Operation within a Circle; then through the middle of G D draw the prolonged line P A: hereon from A set off the Semi-diameter A I equal to AB, by Operat. VI. and from I draw a parallel to G D, by Operat V. which shall cut the prolonged lines A G and A D, at L and K; so is L K the side of a Pentagon described about a Circle: and so lines drawn from L, K, M, and Q, parallel to G R, D M, M P, and R P, shall make the required Pentagon.

Proposition XXVII.

In a given Circle, whose Semi-diameter is A B, *to describe an equilateral Hexagon.*

Operation.

1. On the centre B with the given width, describe a Circle, and set off the width six times in the circumference of the same Circle; as C D, D E, E F, F G, G H, and H C: then draw from B and each Angle

each Angle, the right lines B L, B I, *&c.* and draw from A a Line parallel to G D as A L, and from L the line L I parallel to D E, and so forwards, as by Operat. V. so shall the equilateral Hexagon be described in a Circle.

2. Or make equilateral Triangles on A B and B M, by Operat. III. and the same is done.

Or if you would make an equilateral Triangle within the Circle, then draw from every other Angle of the Hexagon right Lines, and it is performed.

PROPOSITION XXVIII.

In a given Circle, whose Semi-diameter is A M, *to describe an equilateral Polygon of fifteen sides.*

OPERATION.

Describe in the Circle an equilateral Triangle and a Pentagon, (as by Operat. XXVII. and XXV.) so as one Angle of the Triangle, and one Angle of the Pentagon lie in the point A; then shall B G, equal to F C, be one side of the Polygon.

PROPOSITION XXIX.

From a given point G, *without a given Circle, to fit a line, as* K L, *in the same Circle equal to the given right line* D E, *not being longer than the Diameter* H B.

OPERATION.

By Operat. XVIII. inscribe the given line D E in the Circle equal to F G; hereon draw the perpendicular A I into the centre A, by Operat IIII. and draw a right line from the given point C, that shall touch a Circle whose Semi-diameter is A I, and Centre A, by Operat. XV. so shall L K be equal to D E, as was required.

PROPOSITION XXX.

Two Brothers have a triangular piece of Land, as ABC, *wherein they desire to make a Fish-pond, as* DEF, *equal to the Walk about it* A D E B C D F A : *Now they would divide the Fish-pond* D E F, *and the Walk about it, so as the outer and inner bounds of the Walk, as well as the Fish-pond, may be divided into two equal parts by a straight line of Partition running through the Walks that are of an equal breadth. The Question is, how this shall be performed?*

OPERATION.

Describe a Circle in the Triangle A B C, by Operat. XXI. whose Centre shall be at G, and whose Semi-diameter shall be G H : in the middle of this Semi-diameter, as at I, erect the Perpendicular I K equal to G I, by Operat. VIII. then set off (by Operat. VI.) the length of G K from G in G H to P, and at P draw a Parallel to A B to intersect A G and

B G in M and N; from which Points M and N, draw Parallels to A C and B C, as M O and N O, by Operat. V. If now from the mean Proportionals Square made of the ſhorteſt ſide A C, and half of the longeſt ſide A B, we ſubſtract from the Square made of the quarter of the ſides A B, B C, and C A (by the ſecond manner of the X I V. Operat.) and then the product of the Squares ſides added and ſubſtracted from $\frac{AB+BC+CA}{4}$, ſhall A L and A S be produced. Now the right line S L drawn through the points L S, ſhall cut A G (or its prolonged) in a. If then you divide M O and M N in the ſame proportion as A C and A B in S and L, by the ſecond manner of Operat. X. in T and R, and then draw the line T R, which alſo cuts A G in b; and if you ſet off the length b T and b R from a in a S and a L to c and d; and from c and d draw Parallels to A C and A B, to cut A G in D; and then ſet off the length M O and M N from D in the prolongeds D c and D d, to the points F and E. On the Parallels drawn from O and N to M D to cut the prolongeds D c and D d in F and E, and laſtly draw the line E F; ſo is the Propoſition anſwered.

Note,

The reaſon why I here take the ſhorteſt and the longeſt ſide, is, becauſe that this Rule may be fitted to all Triangles, though in ſome Triangles the line of Separation S L may alſo happen to cut through the other ſides: if you do but obſerve, that if to the ſides of the Square (which is the remainder of the

Square

Square made of the fourth part of the ſides A B, B C, and C A, leſs the Square made of the half of its own ſide, and of the other its whole ſide, ſhall be the two ſides that the line of Separation ſhall paſs through) you have added and ſubſtracted from $\frac{AB + BC + CA}{4}$, that the longeſt portion be not longer than one of the ſides that the line of Separation ſhall paſs through. The reſt you may work as before, or otherwiſe; ſo ſhall your Propoſition be anſwered.

Proposition XXXI.

To deſcribe on Paper a Regular Fortification without Arithmetick.

It ſeemed an eaſie matter to the Lord *Matthias Geiger van Baſel* (in his *Artificio Geometrico*) how as well without as with leſs trouble by the Table of Sines to deſcribe and build up a Fortification to an eſpecial advantage for defence. The Rule is thus: Deſcribe a Regular Figure as large as you intend your Fortification to be; then deſcribe upon the half interiour Polygon inwards towards the centre a Quadrat, whoſe Diagonal ſhall be the Face, and its half ſhall give the Flank. If now with your Compaſſes you take off the length of the Diagonal, and place one foot in the middle of the interiour Polygon; ſo ſhall the other foot on the prolonged half Diameter ſhew the point of the Bulwark. Or you may find the Head-line thus: Let fall a Perpendicular from the

the point where the Diagonal-line cuts through the half Diameter on the Line drawn from the centre to the middle of the interiour Polygon; so shall this Perpendicular be the Head-line. If now from the point of half the Face you draw a Perpendicular to the innermost Polygon, it shall give you the Flank, the Gorge, and the Curtain.

By this you must take notice, 1. That the width of the Bulwarks points in great Royal, gives always 80 Rods. 2. The Sight-line is always the half of the Face. 3. That the Flank is half the width of both the Polygons. 4. The Curtain is half of the exteriour Polygon. 5. The Head-line is half the Curtain.

From hence this Author concludes, That because this Proportion remains in all Figures, and agrees with Nature, who affects the Mean, that it is the most certain and most sure manner of Fortification.

Thus far *H. Nottnagel* in his *Manual Architectura Militaris*, fol. 175, and 176.

OPERATION.

By Operat. XVII. describe one half of an Hexagon, whose sides or inward Polygons shall be W C, D X, and D C: On the half of one of these sides, as C F, make a Square, as C F G H, by Operat. I. and XXII. whose Diagonal F H shall cut half the middle line A L in I: from this intersection I let fall a Perpendicular (by Operat. IIII.) on A F, as I K (which is the Head-line:) this added to A C, A D, A X, and A W,

A W, (by Operat. VI.) produces A L A R, A B, and A E. Now drawing lines from each Bulwarks point to the middle of the innermost Polygons, as L F, F R, R Y, B Y, E V, and V L, you have the Faces: through the middle of each, as M, S, *&c.* let fall Perpendiculars to C D, *&c.* as at V T, *&c.* so shall M N, S T, *&c.* be the Flanks; and N C, T D, *&c.* shall be desired Gorges.

Work thus with the other half, so shall the Draft of the first Ground-line be finished.

Note,

The Lord *Geier*'s way is from a Figure of four Angles to twelve Angles, to make the Quadrat on the half of the interiour Polygon; and of a Figure of twelve Angles, and so forwards, he makes Angles of the Bulwarks to be right Angles, or 90 degrees; which happens thus: If from the middle of the interiour Polygon a Perpendicular be let fall upon the half Diameter, measure this length from the Point where it becomes right-angular in the prolonged half Diameter: From thence draw the half interiour Polygon. For the rest, work as before.

PROPOSITION XXXII.

To describe on Paper a Regular Fortification without Arithmetick, and without altering the feet of your Compasses.

OPERATION.

The Lord *Nottnagel*'s words, *fol.* 177, tend to this sense:

ſenſe: If upon a given or elected line, a Regular Figure be deſcribed by the Principles of Geometry and upon the ſides of the Figure with the ſame width of the Compaſſes, be deſcribed a Semi-circle, and the ſame divided into three equal parts in any of the Angles, as here on the ſide A B of the half Hexagon, the Semi-circle B C D E is deſcribed, and without opening the Compaſſes is divided into three equal parts, as B C, C D, and D E: and then with the ſame width of the Compaſſes, from B and C you deſcribe the occult Arches at F, from whence ſtraight lines drawn to A and E, as alſo from D to A and B, and from G to E, ſhall ſhew the lines F A, F E, D A, D B, and G E; then ſhall the line A D be twice interſected, *viz.* at H and I, of which A H ſhall be the Gorge of the aforeſaid Figure, I H the Flank, and A I the Head-line. Thus you may ſee that by this manner of working, it is eaſie to deſcribe a Fortification, and to prick it on Paper, becauſe all Lines are equal. Thus far this Author.

Note,

This Author's purpoſe is only to finde the Head-line A I (which is one third part of A B) the Gorge A H, and the Flank H I, with once opening the Compaſſes to any given or elected interiour Polygon A B. Now I will ſhew how to finde the ſame lines A I, A H, and H I, with once opening the Compaſſes to any given diſtance upon any elected or given interiour Polygon A B. The example ſhall be in a Regular Pentagon, which I will deſcribe and finiſh.

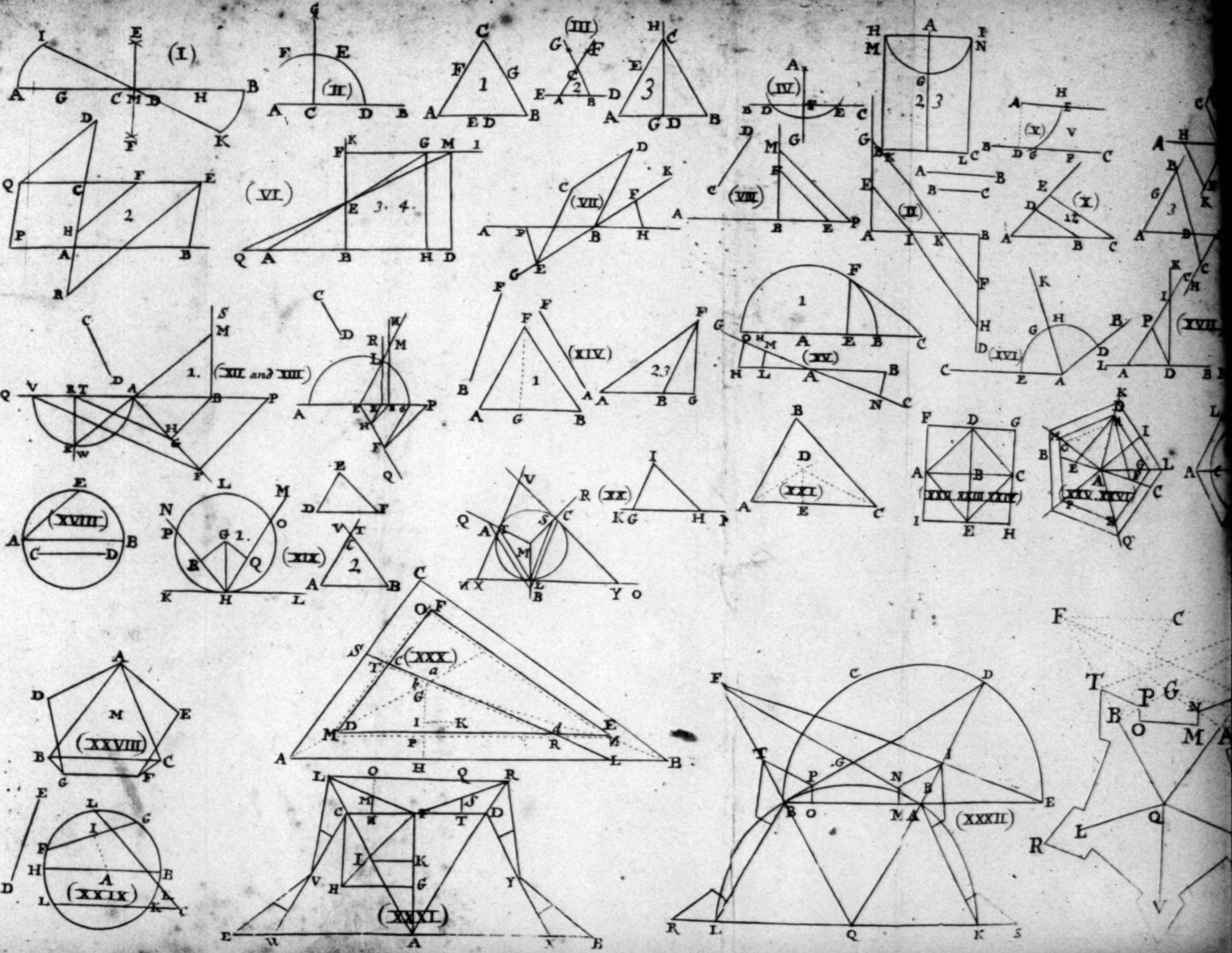
OPERATION.

First finde the sides of the Regular Pentagon by Operat. XXV. as in that figure F D equal to G D: hereon make an equilateral Triangle, as G A D, whereof the upright sides is half of the middle-line, by Operat. XIIII.

On the given interiour Polygon A B, describe the Triangle A B Q, alike in form to the Triangle GAD, by Operat. XIX. (The rest of the Triangles make as in Operat. XXV.) Then make one half of a Regular Hexagon in a Semi-circle, by Operat. XXVII. whose half middle-line is the given interiour Polygon A B; so shall the sides E D, C D, and C B be produced. Hereon describe an equilateral Triangle B C F, by Operat. III. then is what was required brought so far as the Author desires, to finde the intersections on A D, as A I, A H.

To compleat the Figure, joyn the Head-line A I equal to T B, *&c.* (by Operat. VI.) to the Semi-diameters A Q, B Q, *&c.* and cut off the Gorge A H, *&c.* at the ends of the interiour Polygons: Then set off A H (to which A M and B O are equal) from A and from B, it gives M O the Curtain. From M and O, erect perpendicularly the Flanks M N, P O, equal to H I, on A B, by Operat. VIII. Then draw the Sight-lines, as N I, T P, *&c.* and so forwards work all the other Lines; then shall the Figure be compleated to the

E N D.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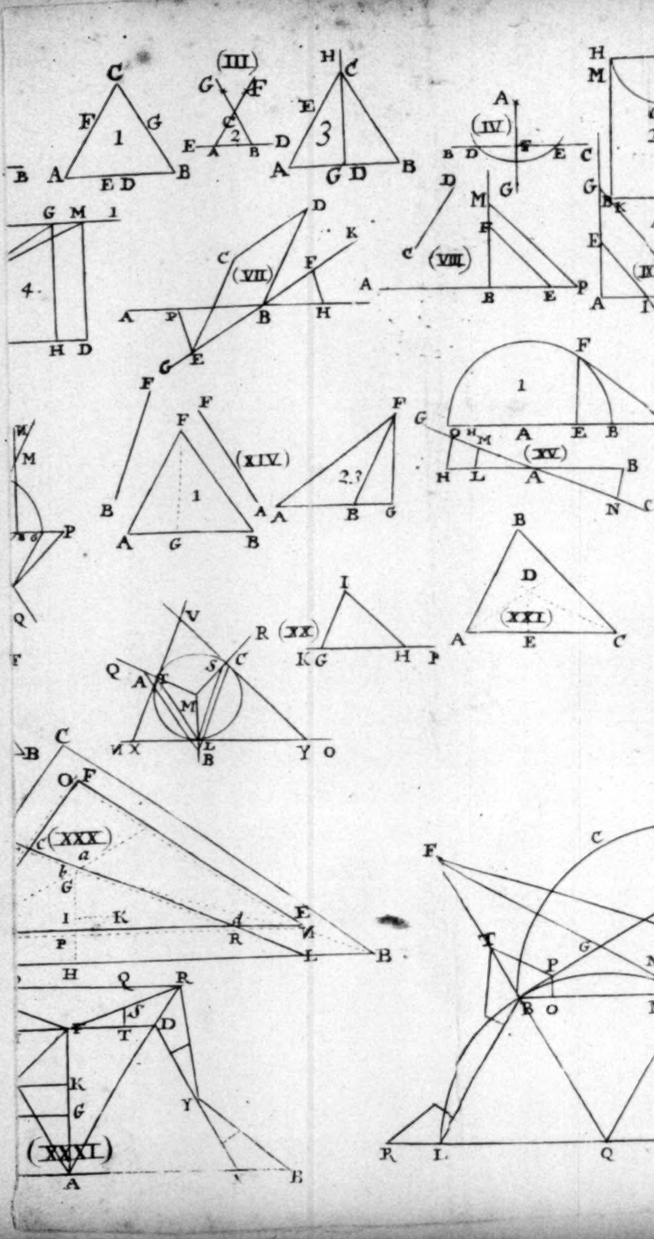
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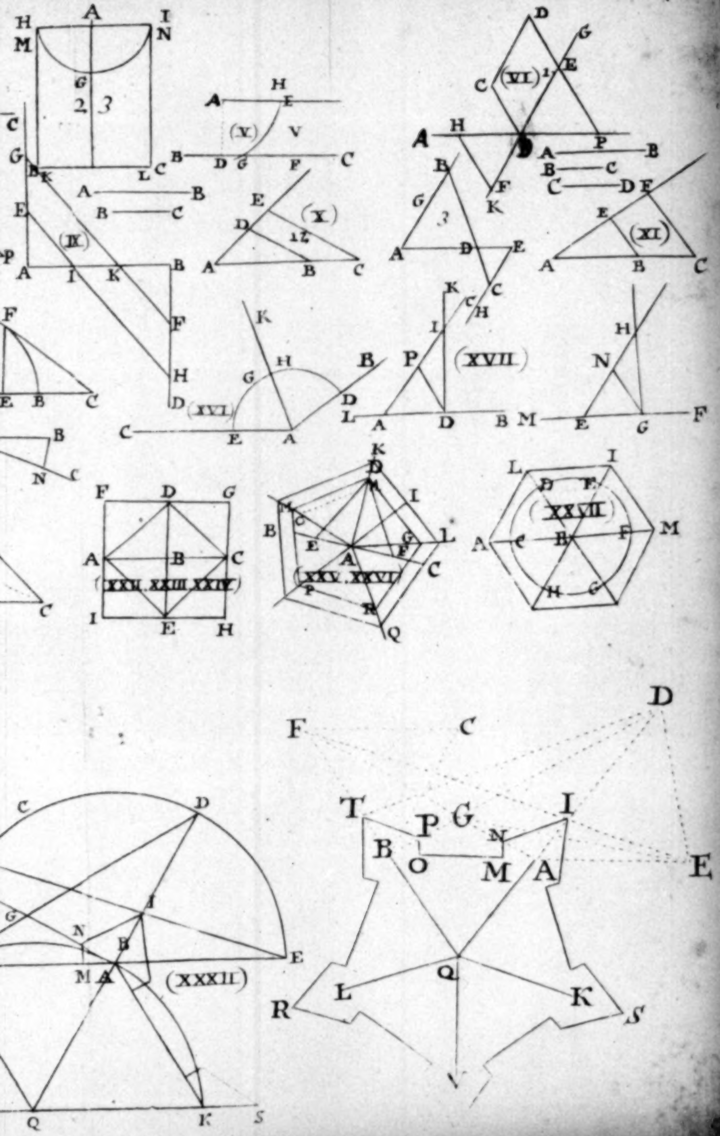

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3

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FINIS.

4

THEOLOGIÆ CHRISTIANÆ PRINCIPIA MATHEMATICA.

AUTORE

JOHANNE CRAIG.

LONDINI,

Typis Johannis Darby, & Impensis Timothei Child ad Insigne Cervi Albi in Cœmeterio D. Pauli.

M. DC. XC. IX.

REVERENDO

IN

CHRISTO PATRI

ET

DOMINO

D[no] GILBERTO,

Providentia Divinâ Episcopo

SARISBURIENSI,

Et Nobilissimi Ordinis à PERISCELIDE

CANCELLARIO,

Tractatum hunc Theologicum

humillime D. D. C. Q.

JO. CRAIG.

PRÆFATIO
AD
LECTOREM.

SCIO quam arduum æquè ac insolitum opus in me suscipiam, cum res à sensibus tam longè remotas ad Leges Geometricas revocare aggredior: Cum vero serio mecum perpenderem quam egregia Scientiarum Naturalium incrementa ex Geometriâ deduxerint & demonstraverint tum veteres tum recentiores Mathematici; fecerunt illa, ut eandem in rebus Theologicis usum aliquem habere posse sperarem: Scientiæ enim adeo Divinæ utilitatem, non ultra angustos vitæ hujus limites extendi posse, absurdum planè videtur; quandoquidem per regulas Geometricas omnia Naturæ opera stabiliantur, ecquis dubitet easdem nos ducere ad sapientissimi Naturæ Autoris cognitionem? quo perfectiùs artem quamlibet intelligimus, eo meliùs nos posse de potentiâ & sapientiâ Artificis determinare ac judicare, certissimum est. Quærenti quid ageret Deus, γεωμετρεῖν τὸν θεὸν *optime respondit Divinus ille Plato Philosophus: & proinde ex Scholis suis Philosophicis omnes Geometriæ ignaros jure merito exclusit. Vana enim est illa Philosophia, quæ nos ad Naturæ ejusque*

Autoris

Autoris cognitionem non deducit; & jejuna admodum est illa utriusque cognitio, quam aliundè quàm ex Geometriâ haurire speramus. Imo tam late patet nobilissimæ hujus Scientiæ utilitas, ut ad Religionis etiam revelatæ, id est, Fidei probabilitatem stabiliendam egregiè conducat; quod in hoc, quem jam in lucem emitto, Tractatu, te L. B. ad plenam tuam satisfactionem reperturum existimo. Generalia quidem solummodo Religionis principia hic tractata videbis, præsertim quæ ad Sacræ Christi *Doctrinæ veritatem, Vitæque futuræ spem nostram stabiliendam conducere existimabam. Idque eo magis necessarium mihi videtur, quanto graviores jam contra Religionis nostræ veritatem sunt Atheorum & Deistarum impetus. Quænam præcipuæ sint grassantis hujus Atheismi causæ non jam sollicitus inquiro: mihi tantum hoc propositum est, ut morbo sævienti remedium aliquod (si possem) afferrem.*

Quosdam fore non dubito, majori ductos zelo quam judicio, qui meos prorsus condemnabunt labores; meque Religionem potius evertere quàm astruere temerè nimis concludent. Illi utique omnia Religionis dogmata tanquam certissima amplectentes, rem Christianismo indignam me præstitisse putabunt, qui ejus probabilitatem tantum evincere conatus fuerim. Illis verò ego nihil jam habeo quod dicam, nisi quod præjudiciis suis præoccupati, Religionis, quam profitentur, fundamenta non accuratè satis hactenus examinaverint; nec Fidei, quæ tantoperè in sacris literis laudatur, naturam ritè intellexerint. Quid enim est Fides? nisi illa mentis persuasio, quâ, propter media ex probabilitate deducta, quasdam propositiones veras esse

esse credimus. Si persuasio ex certitudine oriatur, tum non Fides sed scientia in mente producitur. Sicut enim probabilitas Fidem generat, ita etiam scientiam evertit; & e contrà: Certitudo scientiam simul generat & Fidem destruit. Undè Scientia omnem dubitandi ansam aufert; dum Fides aliquam semper hæsitationem in mente relinquit: & propterea Fides tantis insignitur laudibus, tantaq; sibi annexa præmia habet, quod homines, non obstantibus omnibus illis, quibus premuntur, scrupulis, in recto Virtutis & Pietatis tramite progrediantur; quæq; Creatori suo Omnipotenti grata futura credunt, summâ ope præstare conentur: se tam paratos esse jussis quibuscunq; divinis obsequi offendunt; ut ne ea quidem, quæ probabiliter tantum ab Ipso proveniunt, rejicere velint.

Jamq; non nisi duas alicujus momenti objectiones prævideo, quarum hæc est Prior. Quod non recte definiverim tempus, quo Probabilitas Historiæ Christi evanescere debet; cum novos probabilitatis gradus ex Prophetiarum quarundum completione oriundas non consideraverim; sed eandem in certâ quadam proportione semper decrescentem acceperim. Sed responsio est facilis, Hæc de Christo Servatore Historia non aliter mihi consideranda fuit, quam qualis per aliquot sæcula hactenus transmissa fuit: Si Prophetias illas suam habituras completionem in Calculo meo supponerem, impudenter nimis id quod quæritur postularem. Dein novus ille probabilitatis gradus ex Prophetiarum completione oriturus, non magni erit momenti; nisi pro hominibus istius sæculi, quo eventus prædictioni respondet: tantus certe non erit, ut calculum nostrum perturbare nedum evertere possit.

Poste-

Posterior objectio majorem præ se ferre speciem videtur. Quod scil. Calculus noster Mosis, cæterorumq; V. T. Scriptorum Autoritatem labefactare videtur. Fateor equidem, &, nisi Christi adventus novam illis addidisset probabilitatem, actum jam fuisset ante aliquot secula de eorum Autoritate. In hunc finem venit Filius hominis ut Legem & Prophetas impleret: ut eorum pene evanescentem probabilitatem restitueret. Calculus itaque a me adhibitus Christianismum solidis superstruit fundamentis & Judaismum simul funditùs evertit.

THEO-

THEOLOGIÆ CHRISTIANÆ Principia Mathematica.

DEFINITIONES.

1.

VOluptas eſt ſuavis ille Animi ſenſus, quem objecta Naturæ humanæ convenientia in nobis producunt.

2.

Intenſitas Voluptatis eſt magnitudo ejuſdem ex magnitudine ſuavis iſtius Animi ſensûs determinanda.

3.

Duratio Voluptatis eſt quantitas temporis, per quod ſuavis ille Animi ſenſus in nobis perſeverare ſentitur.

4.

Voluptas æquabilis eſt, quæ eoſdem intenſitatis gradus habet per ſingula durationis ſuæ momenta.

5.

Voluptas uniformiter creſcens eſt illa, cujus intenſitatis gradus creſcunt uniformiter per ſingula durationis ſuæ momenta.

Scholium. Ex infinita varietate incrementorum & decrementorum graduum intenſitatis, aliæ infinitæ voluptatum ſpecies definiuntur.

6.

Probabilitas eſt apparentia convenientiæ vel diſconvenientiæ duarum Idearum per argumenta, quorum connexio non eſt conſtans, aut ſaltem talis eſſe non percipitur.

7.

Probabilitas Naturalis eſt, quæ deducitur ex argumentis propriæ noſtræ obſervationi aut experientiæ conformibus.

8.

Probabilitas Hiſtorica eſt, quæ deducitur ex Teſtimoniis aliorum, qui ſuam affirmant obſervationem aut experientiam.

9.

Suspicio Probabilitatis historicæ est motus Animi in partes historiæ contrarias.

10.

Velocitas Suspicionis est potentia, per quam Animus in aliquo tempore quasi per spatium aliquod in partes historiæ contrarias impellitur.

Scholium. Per spatium hìc intelligo quantitatem Assensûs, quem animus præbet Argumentis Historiæ contrariis. Concipio nimirum Animum ut rem mobilem, & Argumenta ut vires motrices ipsum huc vel illuc impellentes.

AXIOMATA.

1.

Omnis homo conatur voluptatem in Animo suo producere, augere, aut in statu suo voluptatis perseverare.

2.

Conatus Sapientûm sunt in ratione directâ, quam habent veri expectationum suarum valores. Quicunq; hanc Conatuum rationem accuratè sequitur, is est sapientissimus; & qui minùs accurate, minùs sapiens censetur.

3.

Conatus Insipientum sunt in ratione reciprocâ, quam habent veri expectationum suarum valores.

Non intelligendum est hoc axioma in rigore mathematico: hoc solummodo hìc innuere volo, quod illi majores adhibeant conatus ut obtineant expectationem, cujus verus valor est (a) quam ad obtinendam aliam expectationem, cujus verus valor est (na), posito quod n sit major unitate.

HYPOTHESIS.

Omnes homines jus habent æquale ut credantur, nisi contrarium aliundè constiterit. Æquitas hujus suppositionis fundatur in eo, quod res omnes ejusdem Naturæ iisdem præditæ sint qualitatibus naturalibus; sive hæ Animum, sive Corpus respiciunt: estque communi hominum praxi consonum, qui in quibuslibet vitæ hujus Negotiis determinandis hominem quemlibet testem accipiunt, nisi hoc jus suum naturale aliquomodo amiserit.

CAPUT

CAPUT I.

De Probabilitate historicâ quæ viva voce traditur.

EX triplici præsertim capite diminuitur Probabilitas historica. Ex numero Testium, per quos historia successivè transmittitur. Ex distantia loci, ad quem subjectum refertur: sed hoc spectat has solummodo Historias, quarum subjecta principalia sunt permanentia; si enim illa sint transeuntia, nequaquam diminuitur Probabilitas ob loci distantiam. Tertiò ex decursu temporis per quod historia transmittitur. Et juxta quam rationem in horum singulis decrescat probabilitas in sequentibus propositionibus demonstrabitur. Alias causas diminuendi Probabilitates historicas hìc non considero; tum quòd perexiguæ sint, tum maximè quòd vim conclusionum principalium non destruant, & ex principiis hìc positis ad Calculum reduci possint.

Propositio I. *Theorema* I.

Quævis Historia (non contradictoria) ab unius Testis primi testimonio confirmata quendam habet Probabilitatis gradum.

Magna enim Probabilitas componitur ex multis Testium primorum Testimoniis, sicut magnus numerus ex multis unitatibus. Fieri quidem potest, ut talis Probabilitatis Gradus sit tam exiguus, ut illius vim Animus noster vix percipere possit: sicut in motu Corporum, gradus Velocitatis tantillus aliquando est, ut motum oculis discernere nequeamus. Sed (& hic velocitatis, &) iste Probabilitatis gradus est determinatæ magnitudinis, & multoties repetitus Probabilitatem sensibilem producit.

Prop. II. *Theor.* II.

Probabilitas Historica crescit pro numero Testium primorum, qui rem factam enarrant.

Nam Unus Testis unum producit Probabilitatis gradum (per *Prop.* I.) Ergo duo Testes, duos; Tres Testes, tres producunt Probabilitatis gradus, &c. Q. E. D.

Corol. Sit hiſtoria quælibet H, à numero Teſtium primorum $n+m$, homini cuivis A relata, quorum aliqui tantùm, putà n, eandem enarrent homini alteri B: Erit Probabilitas, quam habet A, ad Probabilitatem, quam habet B, ut $n+m$ ad n. Ut ſi (ex gra.) ſit $n=4$, $m=8$, Erit Probabilitas, quam habet A, tripla Probabilitatis, quam habet B.

Prop. III. *Theor.* III.

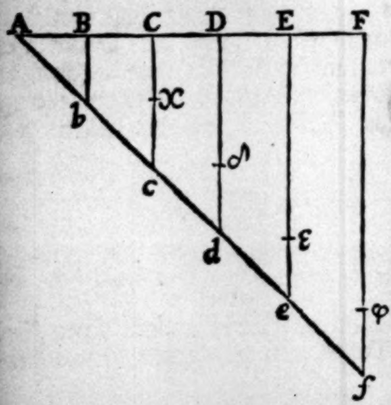
Suſpiciones Probabilitatis hiſtoricæ per unum ſemper Teſtem tranſmiſſæ (cæteris paribus) creſcunt in ratione numerorum Teſtium per quos hiſtoria traditur.

Sit s tota ſuſpicio, quam habemus de fidelitate, cæteriſque Teſtis perfecti virtutibus: Tum ſi unus Teſtis det s, duo Teſtes dabunt 2 s, tres Teſtes dabunt 3 s: & univerſaliter numerus Teſtium n dabit ſuſpiciones n s. Eſt enim s ejuſdem quantitatis in ſingulis (per Hypotheſin).

Corol. Sit M numerus Teſtium, per quos Hiſtoria ſucceſſivè tranſmittitur, erit $\overline{M-1}\times s$ tota ſuſpicio ultimo Teſti tranſmiſſa: habet enim ſuſpiciones omnium Teſtium, præter unam, quæ ex ipſius relatione orietur, & quam proinde ſibi ipſi non tranſmittit.

Prop. IV. *Lemma* I.

Velocitates ſuſpicionis in æqualibus temporibus productæ creſcunt in progreſſione Arithmeticâ.



Sit (in Figura ſequenti) A F tempus per quod Hiſtoria tranſmittitur; Diviſum ſupponatur illud in partes quam minimas & æquales AB, BC, CD, DE, EF. Sitq; linea Bb (ipſi AF normalis) velocitas orta ex temporis ſpatiolo AB; dico, ſub finem temporis ſecundi B C velocitatem ſuſpicionis fore $Cc=2\,Bb$; & quòd ſub finem tertii temporis C D velocitas ſuſpicionis erit $Dd=3\,Bb$: & ſic porro.

Nam Hiſtoria per Tempus B C tranſmiſſa haberet ſuſpicionem

C*x*

Cx (=Bb) etiam cessante omni suspicionis Causâ; Ergo cùm eadem suspicionis causa agit per tempus BC, quæ agebat per tempus AB, augebitur velocitas Cx quantitate xc=Bb, quia tempora sunt æqualia, & causa suspicionis supponitur esse vis uniformis. Adeò ut velocitas suspicionis sub finem temporis secundi BC sit Cc=Cx+xc=2Bb. Eodem modo si in instanti C cessaret omnis suspicionis Causa, progrederetur Historia cum suspicione concepta in C, id est, ad D delatâ, foret ejus velocitas Dδ=Cc=2Bb, Ergo cum eadem uniformis causa suspicionis agit per tempus CD, quæ agebat per tempus æquale AB, augebitur velocitas suspicionis Dδ quantitate δd=Bb; & proinde in instante D, velocitas suspicionis erit Dd=Dδ+δd=3Bb. Similiter, si ad tempus D cessaret omnis Causa suspicionis, progrederetur Historia cum suspicione concepta in D, id est, ad E delata, ejus velocitas esset Eϵ=Dd=3Bb. Ergo cum eadem uniformis Causa suspicionis agit per tempus DE, quæ agebat per tempus æquale AB, augebitur velocitas Eϵ quantitate ϵe=Bb: Adeóque velocitas suspicionis in E, erit Eϵ+ϵe=4Bb. Denique, cessante suspicionis causâ in E, progrederetur Historia cum suspicionis velocitate concepta in E, id est, delata ad F, foret ejus velocitas Fϕ=Ee=4Bb; Ergo cum eadem semper uniformis causa suspicionis agit per tempus ultimum EF, quæ agebat per tempus primum & æquale AB, augebitur velocitas suspicionis Eϕ quantitate ϕf=Bb; & proinde integra velocitas in F erit Ff=Fϕ+ϕf=5B. Unde constat, quòd in temporibus æqualibus AB, BC, CD, &c. Velocitates suspicionis Bb, 2Bb, 3Bb sint in simplici progressione Arithmetica. Q. E. D.

Prop. V. *Theor.* IV.

Suspiciones Probabilitatis historicæ per quodlibet tempus transmissæ (cæteris paribus) crescunt in duplicata ratione Temporum, ab initio Historiæ sumptorum.

Per extremitates b, c, d, e, f linearum Bb, Cc, Dd, Ee, Ff gradus velocitatum denotantium (quæq; lineæ AF tempus designanti normales supponuntur) ducatur linea A b c d e f; adeóque Area figuræ A F f repræsentabit suspicionem productam in tempore A F, sicut & Area A D d repræsentabit suspicionem productam in tempore A D; & sic de aliis. Jam quia Bb, Cc, Dd, &c. sunt in progressione Arithmetica 1, 2, 3, 4, &c. (per *Prop.* IV.) Ideo linea A b c d e f est recta, ut ex Elementis est notum. Ideóq; suspicio producta in tempore AF, est ad suspicionem productam in tempore AD, ut Area

Trianguli

Trianguli A F f ad Aream Trianguli A D d. Sed Triangulum A F f eſt ad Triangulum A D d, ut Quadratum lineæ A F ad Quadratum lineæ A D; (ut conſtat ex Elementis) Ergo ſuſpicio orta in tempore A F eſt ad ſuſpicionem ortam in tempore A D, ut Quadratum lineæ A F ad Quadratum lineæ A D. Id eſt, ſuſpiciones ſunt in duplicata ratione Temporum ab initio hiſtoriæ ſumptorum. Q. E. D.

Corol. Sit tempus per quod Hiſtoria tranſmittitur AF=T, & ſuſpicio inde orta ſit K: Et ſit aliud quodlibet tempus datum AD=t, & cognita ſuſpicio inde orta ſit k: Erit $K=\frac{T^2k}{t^2}$.

Prop. VI. *Theor.* V.

Suſpiciones Probabilitatis hiſtoricæ per quaſlibet diſtantias tranſmiſſæ (cæteris paribus) creſcunt in duplicata ratione diſtantiarum ab initio ſumptarum.

Demonſtratur hæc ut præcedens: Adeóque ſi A ſit locus ad quem Hiſtoriæ ſubjectum refertur, ſitq; illa delata ad diſtantiam quamlibet AF=D, ex qua oriatur ſuſpicio Q, & ex quavis alia diſtantia cognita AD=d orta ſit ſuſpicio quædam cognita quæ erit $Q=\frac{D^2q}{d^2}$.

Prop. VII. *Problema* I.

Quantitatem Probabilitatis Hiſtoriæ cujuſvis H, per unum ſemper Teſtem tranſmiſſæ, determinare.

Deſignet x probabilitatem integram, quam primus Teſtis ſecundo tranſmittere poteſt, poſt temporis & loci intervalla quàm minima; ſitq; M (vel m) numerus Teſtium, per quos in dato tempore T, & ad diſtantiam datam D, Hiſtoria illa H tranſmittitur; ſit s ſuſpicio cognitæ quantitatis, quam ex uno quovis Teſte ortam ſupponimus: k ſuſpicio cognita, quam ex dato quovis tempore t; & q ſuſpicio etiam cognita, quam ex data quavis diſtantia d, ortas ſupponimus. Sitq; P probabilitas quæſita. Erit $P=x+\overline{M-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}$.

Nam $\overline{M-1}\times s$ eſt tota ſuſpicio, quæ oritur ex numero Teſtium (per *Prop.* III.): Et $\frac{T^2k}{t^2}$ eſt tota ſuſpicio orta ex tempore dato T (per

Prop. V.) & $\frac{D^2q}{d^2}$ est tota suspicio orta ex distantia D (per *Prop.* VI.) Ergo $\overline{M-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}$ est suspicio integra, quæ historiam H, per numerum Testium M, post elapsum tempus T, & ad distantiam D, transmissam afficiet: &, cùm x sit tota ejus sub initio probabilitas ex hypothesi; patet $P=x+\overline{M-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}$. Q. E. D.

Scholium. Quia per x designatur tota Probabilitas, quam primus Testis secundo transmittit; ideo in numero per M denotato, ipse primus Testis non includitur; quem proinde in sequentibus nomine Historici à cæteris distinguemus. Per Testes intelligemus eos omnes, qui suam Historiæ cognitionem ex Historici observatione aut experientia deducunt.

Exemplum. Quantam probabilitatem habet decimus Testis historiæ H, post elapsum tempus 10t, & ad distantiam 8d: quia in hoc casu $M=10$, $T=10t$, $D=8d$; ideo per Canonem præcedentem, $P=x+9s+100k+64q$ est probabilitas, quam habet decimus ille Testis.

Notandum verò est, quod x, s, t, k, d, q sint quantitates cognitæ; sunt enim totidem unitates ad mensurandam Probabilitatem necessariæ; quæ proinde (sicut in omni alio mensurationis genere) ad arbitrium mensurantis assumi possunt.

Prop. VIII. *Prob.* II.

Datis, numero serierum Testium, numero Historicorum Testi primo uniuscujusq; seriei Historiam transmittentium, item temporibus & distantiis per quæ Historia aliqua H, ad hominem quemlibet A, transmittitur: invenire quantitatem Probabilitatis, quam habet A de veritate Historiæ H.

Inveniatur Probabilitas ab unaquâq; serie transmissa separatim (per *Prop.* VII.) Erítq; summa harum omnium Probabilitas quæsita, quæ homini A ab omnibus transmittitur.

Exemplum. Habeat A Historiam H à duabus Testium seriebus derivatam. Sitq; b numerus Historicorum, m numerus Testium, T tempus, D distantia, per quæ in prima serie Historia transmittitur. Item c numerus Historicorum, n numerus Testium, G tempus, L distantia, per quæ in secunda Testium serie Historia H ad A transmittitur. Jam

bx eſt Probabilitas, quam habet primus Teſtis in prima ſerie, & cx eſt probabilitas, quam habet primus Teſtis ſecundæ ſeriei (per *Prop.* II.) Ergo (per *Prop.* VII.) invenietur.

$$\left.\begin{array}{l} bx+\overline{m-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2} \\ \\ cx+\overline{n-1}\times s+\frac{G^2k}{t^2}+\frac{L^2q}{d^2} \end{array}\right\}$$

Probabilitas tranſmiſſa ad A per primam ſeriem.

Probabilitas tranſmiſſa ad A per ſecundam ſeriem.

Addantur hæ duæ Probabilitates & ſumma utriuſque ſc.

$bx+cx+\overline{m-1+n-1}\times s+\frac{T^2+G^2}{t^2}k+\frac{D^2+L^2}{d^2}q$ Erit integra Probabilitas tranſmiſſa ad A ab utraq; ſerie.

Corol. Si numerus hiſtoricorum b, numerus Teſtium (m) tempus T, & diſtantia D ſint eadem in omnibus Teſtium ſeriebus, quarum numerus ſit (a) tum erit Probabilitas quæſita

$$P=a\times\overline{bx+\overline{m-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}}.$$

Et hunc caſum præcipuè reſpicio in ſequentibus, niſi aliter expreſſè moneatur.

Prop. IX. *Prob.* III.

Data quavis hiſtoria H; aliam hiſtoriam h invenire, quæ, ſi à dato numero ſerierum teſtium (a) tranſmittatur, habeat Probabilitatem in ratione data ad probabilitatem hiſtoriæ datæ H.

Sit e numerus ſerierum Teſtium, c numerus Hiſtoricorum, n numerus Teſtium, G tempus, & L diſtantia, per quæ Hiſtoria data H tranſmittitur; a, b, m, T, D eaſdem reſpectivè denotent quantitates in hiſtoria quæſita h; ſitq; ratio data r ad 1. Jam ex datis e, c, n, G, L, a, inveniendæ ſunt b, m, T, D, hoc modo. $ecx+\overline{en-e}\times s+\frac{eG^2k}{t^2}+\frac{eL^2q}{d^2}$ eſt probabilitas hiſtoriæ datæ H, (per *Prop.* VIII. *Corol.*) Et $abx+\overline{am-a}\times s+\frac{aT^2k}{t^2}+\frac{aD^2q}{d^2}$ eſt probabilitas hiſtoriæ inveniendæ h (per *Corol. Prop.* VIII. Ergo erit

$$abx+\overline{am-a}\times s+\frac{aT^2k}{t^2}+\frac{aD^2q}{d^2}:ecx+\overline{en-e}\times s+\frac{eG^2k}{t^2}+\frac{eL^2q}{d^2}::r:1$$ Ex

condi-

conditione problematis. Quare multiplicando terminos medios & extremos erit $ab x + \overline{am - a} \times s + \frac{aT^2k}{t^2} + \frac{aD^2q}{d^2} = rec x + \overline{ren - re} \times s + \frac{reG^2q}{t^2} + \frac{reL^2q}{d^2}$. Fiat comparatio inter terminos homologos, erítq; $ab = rec$. $am - a = ren - re$. $aT^2 = reG^2$. Atque $aD^2 = reL^2$, quæ æquationes reductæ dabunt $b = \frac{rec}{a}$. $m = \frac{ren - re + a}{a}$. $T = \sqrt{\frac{reG^2}{a}}$. $D = \sqrt{\frac{reL^2}{a}}$. Q. E. I.

Scholium. Data vel inventa dicitur Hiſtoria, cùm dantur vel inveniuntur numerus ſerierum Teſtium, numerus Hiſtoricorum, numerus Teſtium, diſtantia & Tempus per quæ tranſmiſſa vel tranſmittenda eſt Hiſtoria.

Prop. X. *Prob.* IV.

Data quavis hiſtoria H, aliam hiſtoriam h invenire; quæ, ſi à dato numero Hiſtoricorum b tranſmittatur, habeat probabilitatem in ratione data ad probabilitatem Hiſtoriæ datæ H.

Deſignatis quantitatibus ut in propoſitione præcedenti erit $\overline{a \times bx + \overline{m - 1} \times s + \frac{T^2k}{t^2} + \frac{D^2q}{d^2}} = \overline{re \times cx + \overline{n - 1} \times s + \frac{G^2k}{t^2} + \frac{L^2q}{d^2}}$.

Unde, reductis æquationibus ex comparatione terminorum oriundis, erit $a = \frac{rec}{b}$ numerus ſerierum Teſtium, & $m = \frac{nb - b + c}{c}$ numerus Teſtium, $T = \sqrt{\frac{bG^2}{c}}$ tempus; $D = \sqrt{\frac{bL^2}{c}}$ diſtantia, per quæ Hiſtoria illa h invenienda tranſmitti debet, ut ejus probabilitas ſit ad probabilitatem Hiſtoriæ datæ H, ut r ad 1.

Prop. XI. *Prob.* V.

Datâ quavis hiſtoriâ H, aliam hiſtoriam h invenire, quæ, ſi ad Teſtem in dato ordine per (m) deſignato exiſtentem, tranſmittatur, habeat probabilitatem in ratione data ad probabilitatem hiſtoriæ datæ H.

Designentur quantitates ut prius, tum ex datis e, c, n, G, L, r, m, inveniendæ sunt a, b, T, D. Factâ autem reductione æquationum, quæ proveniunt ex comparatione terminorum (ut in *Prop.* IX.) erit $a=\frac{ren-re}{m-1}$ numerus serierum Testium: $b=\frac{cm-c}{n-1}$ numerus Historicorum: $T=\sqrt{\frac{\overline{m-1}\times G^2}{n-1}}$ tempus: $D=L\times\sqrt{\frac{m-1}{n-1}}$ distantia, per quæ Historia invenienda h transmitti debet.

Exemplum. Transmittatur historia data H ad quartum Testem à duabus Testium seriebus, & tribus historicis post 100 annos, & ad distantiam 1000 milliarium; quæritur historia, quæ ad quintum Testem delata habeat probabilitatem duplam probabilitatis historiæ datæ: In hoc exemplo, $e=2$, $c=3$, $n=4$, $G=100$, $L=1000$, $m=5$, $r=2$. Substitutis his valoribus, in quantitatibus modò inventis, erit $a=3$, $b=4$, $T=200\sqrt{\frac{1}{3}}$. $D=2000\sqrt{\frac{1}{3}}$.

Unde constat, quòd historia h, à quatuor historicis, per tres Testium series, post elapsos annos $200\sqrt{\frac{1}{3}}$, & ad distantiam $2000\sqrt{\frac{1}{3}}$ milliarium ad quintum Testem delata, erit duplo probabilior historiâ datâ H.

Prop. XII. *Prob.* VI.

Data quavis historia H, aliam historiam h invenire, quæ post datum tempus T habeat probabilitatem in ratione data ad probabilitatem historiæ H.

Reducantur æquationes ex comparatione terminorum provenientes, & invenientur: numerus serierum Testium scil. $a=\frac{reG^2}{T^2}$. Numerus Historicorum sc. $b=\frac{cT^2}{G^2}$. Numerus Testium sc. $m=\frac{nT^2-T^2}{G^2}+1$. Et distantia sc. $D=\frac{TL}{G}$: per quæ historia invenienda h est transmittenda. Cùm ergo datur T, & inveniuntur a, b, m, D, ideo inventa est historia h (per *Schol. Prop.* IX.)

Prop.

Prop. XIII. *Prob.* VII.

Datâ quavis hiſtoria H, aliam hiſtoriam h (cujus ſubjectum ad locum refertur) invenire, quæ ad diſtantiam datam D tranſmiſſa habeat probabilitatem in ratione data ad probabilitatem hiſtoriæ datæ H.

Reductis æquationibus ex terminorum comparatione provenientibus, invenies, numerum ſerierum Teſtium ſcil. $a=\frac{reL^2}{D^2}$: Numerum hiſtoricorum $b=\frac{cD^2}{L^2}$: numerum Teſtium $m=\frac{nD^2-D^2}{L^2}+1$. Tempus $T=\frac{GD}{L}$. Q. E. I.

Scholium. Si in quovis horum problematum caſu a, b vel m ſint numeri fracti; capiantur numeri integri fractis hiſce proximi.

Prop. XIV. *Theor.* VI.

Probabilitas hiſtorica ab uno hiſtorico, & per unam tantùm Teſtium ſeriem tranſmiſſa, quamvis continuò decreſcat, in nullo tamen tempore dato penitùs evaneſcit.

Jam $x+\overline{m-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}$ eſt probabilitas ab uno Hiſtorico, & per unam Teſtium ſeriem, tranſmiſſa (per *Prop.* VII.) At ſi hæc in quovis tempore dato T evaneſcere poſſet, foret in hoc caſu $x+\overline{m-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}=0$. Sed ſi hoc in quovis caſu fieri poſſet, tum poſſibile etiam eſt, ut in illo caſu $a\times\overline{x+m-1\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}}=0$, utcunque magnus ſupponatur numerus Serierum Teſtium (a). Quod falſum eſt, tantus enim aſſumi poteſt numerus (a), ut ſub initio hiſtoriæ, ejus probabilitas ſit major quavis probabilitate data ab uno hiſtorico producta; ſed probabilitas quavis data major in nullo tempore dato evaneſcit: Ergo impoſſibile eſt ut (cùm ſumitur numerus (a) permagnus) $a\times\overline{x+\overline{m-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}}=0$. Ergo impoſſibile eſt, ut $x+\overline{m-1}\times s+\frac{T^2k}{t^2}+\frac{D^2q}{d^2}=0$. Nam ſi productum ex multiplicatione

 duarum

duarum quarumlibet quantitatum ſit majus nihilo, oportet etiam ut ſinguli factores ſeorſim ſumpti ſint nihilo majores. Unde conſtat propoſitum.

Scholium. Quamvis nunquam penitus evaneſcat probabilitas Hiſtorica, tamen in progreſſu temporis tam exigua redditur, ut illius vim Animus vix percipere poſſit. Hoc itaq; ſupereſt, ut oſtendatur Methodus determinandi tempus, quo evaneſcit gradus ille probabilitatis, qui ad vim in Animo ſenſibilem producendam eſt neceſſarius. Utq; hoc ſimpliciſſimè fiat, ſequentes facio Hypotheſes, quas à vero non multùm remotas eſſe exiſtimo. (1.) Quòd $s = -\frac{x}{10}$, id eſt, Probabilitas integra ab Hiſtorico, primo Teſti tranſmiſſa, nullam ſenſibilem vim producit (cæteris paribus) in Animo Teſtis undecimi. (2.) Quòd ſpatio annorum $50 = t$, oriatur ſuſpicio $k = -\frac{x}{100}$, id eſt, Quòd Teſtis primus, qui, ſi ſtatim Hiſtoriam alicui tradat, ac eandem ab ipſo Hiſtorico accipit, decimam tantùm partem probabilitatis ſibi tranſmiſſæ deſtrueret (per *Hyp.* I.) ſi relationem ſuam per 50 annos procraſtinet (præter illam decimam partem) centeſimam probabilitatis ſibi tranſmiſſæ partem deſtruet. (3.) Quòd ad diſtantiam milliarium $50 = d$ oriatur ſuſpicio $q = -\frac{x}{10000}$ in hiſtoriis, quarum ſubjecta ad locum permanentem referuntur. (4.) Quòd vita unius cujuſlibet Teſtis poſſit per annos $50 = t$ durare. Adeóq; (5.) Erit numerus Teſtium, per quos Hiſtoria per quodlibet tempus T tranſmiſſa $m = \frac{T}{t}$: ſequitur hæc ex quarta Hypotheſi.

Prop. XV. *Prob.* VIII.

Quando evaneſcet probabilitas cujuſvis Hiſtoriæ (cujus ſubjectum eſt tranſiens) vivâ tantum voce tranſmiſſæ, determinare.

Evaneſcet illa, quando $bx + \overline{m-1} \times s + \frac{T^2 k}{t^2} = 0$ (per *Prop.* VIII.) ubi b eſt numerus Hiſtoricorum, m numerus Teſtium, & T tempus evaneſcentis probabilitatis quæſitum. Jam quia $m = \frac{T}{t}$, $s = -\frac{x}{10}$, $k = -\frac{x}{100}$ (per *Hyp.* 5, 1, 2.) ideo ſubſtituantur hi valores quantitatum

tum m, s, k; & erit $bx - \frac{xT}{10\,t} + \frac{x}{10} - \frac{xT^2}{100\,t^2} = 0$. Reducatur hæc æquatio per vulgares Algebræ Regulas, & invenies.

$$T = t\sqrt{100\,b + 35} - 5\,t.$$

Tempus, quando Historiæ cujuslibet probabilitas evanescit.

Corol. Evanuit probabilitas Historiæ Christi, sub finem sæculi octavi, in quantum illa à Traditione tantum orali dependet. Nam in hoc casu $b = 4$, unde $\sqrt{100\,b + 35} = \sqrt{435} = 21$ ferè; Ergo per Canonem hujus propositionis $T = 21\,t - 5\,t = 16\,t$, sed $t = 50$ annis (per *Hyp.* 2.) Ergo $T = 16\,t = 800$ annis. Q. E. D.

Scholium. Eodem modo invenias tempus, quando evanescet probabilitas cujusvis Historiæ, cujus subjectum est permanens: Summopere tamen cavendum est, ne k eandem habere quantitatem capias in historiis, quarum subjecta sunt permanentia, quam habet in Historiis, quarum subjecta sunt transeuntia: Est enim longè major in his, quam in illis: Et quia (per *Hyp.* 2.) supposuimus $k = -\frac{x}{100}$ in Historiis materiam transeuntem transmittentibus, fieri potest, ut, in Historiis subjecta permanentia tradentibus, $k = -\frac{x}{50}$, vel etiam minor, dummodo subjecta illa permanentia sint in loco accessibili; si enim sint in loco inaccessibili, eodem modo tractandæ sunt earum probabilitates, ac si subjecta essent transeuntia.

CAPUT

CAPUT II.

De Probabilitate Hiſtoricâ, quæ per Teſtimonia ſcripta tranſmittitur.

Prop. XVI. *Prob.* IX.

QUantitatem Probabilitatis hiſtoricæ ab uno Hiſtorico primo literis conſignatæ determinare.

Sit z integra probabilitas Hiſtoriæ ſub initio publicationis primi exemplaris, n numerus exemplarium, T tempus, & D diſtantia Loci, per quæ Hiſtoria ſcripta tranſmittitur. Sitq; f ſuſpicio orta ex tranſcriptione alicujus Exemplaris (eſt verò f in omnibus eadem, quia Tranſcriptores ſunt pariter fideles, per Hypotheſin) cæteris poſitis, ut in capite præcedenti, erit quæſita probabilitas

$$P = z + \overline{n - 1 \times f} + \frac{T^2 k}{t^2} + \frac{D^2 q}{d^2}:$$

Corol. Sit c numerus Hiſtoricorum primorum, c numerus exemplarium ſecundorum per totidem ſeries Hiſtoriam tranſmittentium, poſito quòd ſingulæ Exemplarium ſeries ex uno tantùm Exemplari ſecundo traducantur, erit Hiſtoriæ ſic tranſmiſſæ probabilitas

$$P = \overline{r \times cz + \overline{n - 1} \times f + \frac{T^2 k}{t^2} + \frac{D^2 q}{d^2}}:$$

Scholium. Per Hiſtoricos primos intelligo eos, qui Hiſtoriæ cognitionem ex propria obſervatione aut experientia deducunt. Et per Exemplar primum intelligo (non unum tantum, ſed) quotlibet exemplaria ab ipſo primo Hiſtorico ſcripta vel impreſſa. Jam quia Hiſtoria ſcripta majorem longè probabilitatem habet, quàm Hiſtoria per vivam vocem tradita; & quia hujus probabilitas nunquam evaneſcit (per *Prop.* XIV.) ſequitur illius etiam probabilitatem in nullo tempore dato penitus evaneſcere. Attamen cum continuò decreſcat, neceſſe eſt ut tandem etiam illa perexigua reddatur; Ut ergo determinetur tempus, quo perit hæc ſenſibilis probabilitas fiant ſequentes hypotheſes.

(1.) Quod

(1.) Quòd $z=10x$, id eſt, Hiſtoricus tranſmittit decies plures probabilitatis gradus, quando teſtimonium ſuum per ſcripta, quàm cùm per vivam tantùm vocem illud traditur. (2.) Quòd $t=\frac{s}{10}$, id eſt, ſuſpicio fidelis tranſcriptoris eſt decima tantùm pars ſuſpicionis fidelis Teſtis, unde ſequitur quòd $f=-\frac{x}{100}$ (per *Hyp.* 1. *Prop.* XIV. & *Hyp.* 2. hujus). (3.) Quòd Exemplar Hiſtoriæ poſſit per annos $200=4t$ durare. Unde ſequitur (4.) quòd numerus Exemplarium, Hiſtoriam per quodlibet tempus T tranſmittentum, ſit $n=\frac{T}{4t}$.

Prop. XVII. *Prob.* X.

Quantitatem præſentis probabilitatis Hiſtoriæ Chriſti à quatuor Hiſtoricis ſcriptæ, & per unam Exemplarium ſeriem tranſmiſſæ, determinare.

Præſens probabilitas Hiſtoriæ Chriſti eſt $cz+\overline{n-1}\times f+\frac{T^2k}{t^2}$ (per *Corol. Prop.* XVI.) ſed in hoc caſu numerus primorum Hiſtoricorum eſt $c=4$, $T=1696$ annis $=34t$. Et (per Hypotheſes *Prop.* XVI.) $z=10x$, $n=\frac{34}{4}$, $f=-\frac{x}{100}$, $k=-\frac{x}{100}$ ſubſtituantur hi valores quantitatum c, n, f, T, & erit $p=40x-\frac{34x}{400}-\frac{1156x}{200}$ præſens probabilitas Hiſtoriæ Chriſti, quæ reducta dat $p=\frac{11342x}{400}$; id eſt quàm proximè $p=28x$. Tanta itaq; eſt præſens probabilitas Hiſtoriæ Chriſti, quantam habuiſſet ille, qui (ipſius Chriſti temporibus) vivâ tantum voce eandem à 28 Diſcipulis Chriſti acciperet. Q. E. I.

Prop. XVIII. *Prob.* XI.

Temporis ſpatium definire, in quo Hiſtoriæ Chriſti ſcriptæ probabilitas evaneſcet.

Evaneſcet illa quando $cz+\overline{n-1}\times f+\frac{T^2k}{t^2}=0$ (per *Corol. Prop.* XVI.) hoc eſt (ſubſtitutis valoribus quantitatum z, n, f, k, nec non

* $4=c$)

4 = c) quando $40 + \frac{1}{100} - \frac{T}{400\,t} - \frac{T^2}{100\,t^2} = 0$: Hæc æquatio reducta dabit Tempus evaneſcentis probabilitatis quæſitum, ſcil. $T = t\sqrt{4001 + \frac{1}{64}} - \frac{1}{8}t$; vel potius (neglectis fractis, quod in hujuſmodi computationibus ſine magno erroris periculo fieri poſſit) erit tempus quæſitum $T = t\sqrt{4001}$, hoc eſt, quia $\sqrt{4001} = 63$ & $t = 50$ annis, $T = 3150$ annis: unde conſtat, quòd poſt annos 3150 à nativitate Chriſti, evaneſcet hiſtoriæ ejus ſcriptæ probabilitas. Q. E. I.

Corol. Neceſſe eſt, ut Chriſtus veniat, antequam elabantur anni 1454. Nam neceſſe eſt, ut Chriſtus veniat, priuſquam evaneſcat Hiſtoriæ ſuæ probabilitas; ſed illa peribit, elapſis à noſtro tempore annis $1454 = 3150 - 1696$; Ergo neceſſe eſt, ut veniat antequam elabantur anni 1454 à noſtro tempore. Q. E. D. Et in nullo tempore minori annis 1454 neceſſe eſt ut veniat, in quantum ejus adventus ex defectu probabilitatis ſuæ hiſtoriæ dependet: Et certè multa me movent, ut ſuſpicer, illum non priùs venturum, quàm ferè evanuerit hiſtoriæ ſuæ probabilitas; hoc enim diſertè innuere videtur Lucas in Hiſtoriæ ſuæ Capite 18, verſu 8, ubi Chriſtum expoſtulâſſe narrat in hunc modum —Nihilominus, cùm venerit Filius hominis, an fidem in Terrâ inveniet—Tantilla ſcil. ad adventum Chriſti erit Hiſtoriæ ſuæ probabilitas, ut dubitet, an quenquam reperturus ſit, qui huic de ipſo Hiſtoriæ fidem adhibebit. Unde patet, quàm graviter errent illi omnes qui Chriſti adventum ad noſtra tempora tam propè conſtituunt.

Prop. XIX. *Prob.* XII.

Quantitatem Probabilitatis cujuſvis Hiſtoriæ ſcriptæ determinare, juxta aſſumptas Hypotheſes.

Cujuſvis Hiſtoriæ ſcriptæ (cujus ſubjectum eſt tranſiens) probabilitas eſt $p = r \times \overline{cz + \overline{n-1} \times f + \frac{T^2 k}{t^2}}$ (per *Corol. Prop.* XVI.) Ergo (ſubſtitutis valoribus quantitatum z, n, f, k) erit

$$p = r x \times \frac{4000\,ct^2 - Tt + 4\,tt - 4\,TT}{400\,t^2}.$$

Prop.

Prop. XX. *Prob.* XIII.

Temporis ſpatium definire, in quo Hiſtoriæ cujuſvis ſcriptæ probabilitas evaneſcet.

Evaneſcet illa quando $cz + \overline{n-1 \times f} + \frac{T^2 k}{t^2} = 0$ (per *Corol. Prop.* 16.) hoc eſt (ſubſtitutis valoribus quantitatum z, n, f, k) quando $20c + \frac{1}{100} - \frac{T}{400t} - \frac{T^2}{100t^2} = 0$; hæc æquatio reducta dabit tempus evaneſcentis probabilitatis quæſitum, ſcil. $T = t\sqrt{1000c + \frac{65}{64}} - \frac{1}{8}t$. vel neglectis fractis $T = t\sqrt{1000c + 1}$:

Datur t ſcil. ſpatium 50 annorum, & c numerus Hiſtoricorum primorum; Ergo habetur T tempus quæſitum, in quo evaneſcit Hiſtoriæ probabilitas.

Scholium. Eodem modo ſolvuntur duo poſtrema Problemata, quando Hiſtoriæ reſpiciunt ſubjecta in loco acceſſibili conſtituta.

Prop. XXI. *Prob.* XIV.

Determinare quantitatem probabilitatis Hiſtoriæ, quæ partim vivâ voce, partim per ſcripta tranſmittitur.

Inveniatur probabilitas ejus pro temporis ſpatio, quo vivâ voce traditur (per *Prop.* VIII.) & probabilitas ejus pro temporis ſpatio, quo per ſcripta traditur (per *Prop.* XVI.) dabit utriuſq; ſumma probabilitatem quæſitam. Q. E. I.

Prop. XXII. *Prob.* XV.

Datis duabus ejuſdem rei Hiſtoriis contrariis, utra harum ſit probabilior, quantáq; ejus ſit probabilitas, determinare.

Inveniatur probabilitas utriuſq; (per *Propp.* VIII. & XVI.) & ſubſtituantur valores quantitatum m, s, k, q, z, n, f, & ſtatim conſtabit utra ſit probabilior; ſubducatur probabilitas minor ex majori, & reliqua erit tota probabilitas Hiſtoriæ probabilioris. Q. E. I.

Conclusio. Credo me jam ea omnia dilucidè satis explicuisse, quæ ad probabilitatis Historicæ determinationem sunt necessaria. Ad alteram jam materiæ meæ partem progredior, scil. ad definiendas Voluptatum Quantitates. Voluptas enim est unicum omnium actionum & conatuum nostrorum principium. Quicquid agimus aut patimur, quicquid cupimus aut aversamur, omnia tamen Voluptatis causâ fieri pro certo constat. Ut ergo homines prudenter suas prosequantur voluptates, necesse est, ut earum quantitates & valores accuratè determinare queant: quod proinde in sequentibus docebitur.

CAPUT III.

De Voluptate Æquabili.

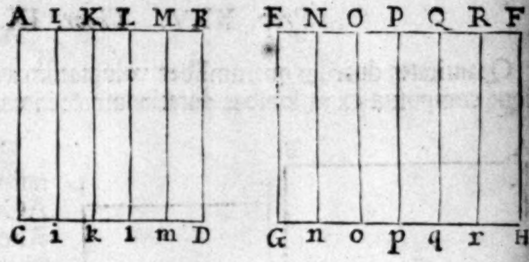
UT sequentes propositiones faciliùs demonstrentur, Durationem Voluptatis per lineam rectam designo, & gradus ejus intensitatis per lineas rectas ad singula puncta prioris lineæ perpendiculares: & proinde si ducta intelligatur linea per alteras extremitates horum perpendiculorum, formabitur inde Figura plana, quæ Voluptatis illius Quantitatem optimè repræsentabit: Adeóque ex figuræ hujus proprietatibus facile erit voluptatis illius proprietates deducere.

Prop. XXIII. *Theor.* VII.

Quantitates duarum Voluptatum æquabilium (quarum Intensitates sunt æquales) sunt in ratione Durationum directâ.

Sit

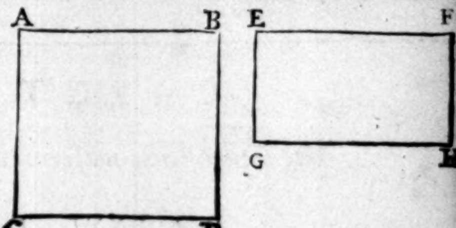
Sit AB duratio unius Voluptatis, ejus quantitas V. sintque AC, Ii, Kk, Ll, &c. gradus intensitatis in diversis durationis Instantibus. Et esto EF duratio alterius voluptatis, cujus quantitas V, & gradus intensitatis EG, Nn, Oo, &c. In Instantibus E, N, O, &c. Jam cum utraq; voluptas sit æquabilis (ex Hypothesi) ideo linea ducta per C, i, k, l, m, D, & linea ducta per G, n, o, p, q, r, H, sunt rectæ ad AB & EF parallelæ (per Def. 4.) Ergo figuræ ABCD EFGH designantes utriusq; Voluptatis Quantitatem sunt parallelogramma rectangula. Jam v=AB×AC, V=EF×EG (ex Elementis) & AC = EG (ex hypothesi propositionis) Ergo V = EF × AC. Unde v. V :: AB × AC. EF × AC. sed (ex Elementis) AB × AC. EF × AC :: AB. EF. Ergo v. V :: AB. EF. Quod erat demonstrandum.



Prop. XXIV. *Theor.* VIII.

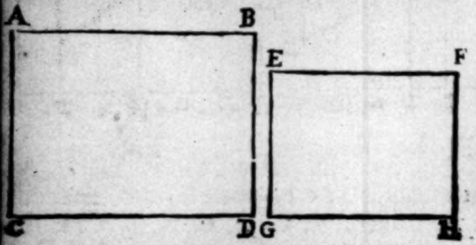
Quantitates duarum voluptatum æquabilium (quarum tempora durationum sunt æqualia) sunt in ratione directa intensitatum.

Sit prioris voluptatis duratio AB, & constans ejus intensitatis gradus AC, quantitas integra sit v. Nec non alterius Voluptatis duratio EF, intensitas constans EG,& quantitas V.Erit v=AB×AC, V=EF×EG, sed ex hypothesi AB=EF, ergo V = AB ×EG. Unde v. V :: AB×AC. AB×EG. sed AB×AC. AB×EG :: AC. EG. Ergo v. V :: AC.EG. Quod erat demonstrandum.



Prop. XXV. *Theor.* IX.

Quantitates duarum quarumlibet voluptatum æquabilium sunt in ratione composita ex rationibus durationum & intensitatum directis.



Sit unius voluptatis duratio AB$=$r, intensitas AC$=$n, & quantitas v. Alterius autem voluptatis sit quantitas V, duratio EF$=$s, intensitas EG$=$m; Jam quia Figuræ, quæ repræsentant quantitates voluptatum æquabilium sunt Parallelogramma rectangula, ideo v$=$rn, V$=$sm, ideo $\frac{V}{v} = \frac{sm}{rn} = \frac{s}{r} \times \frac{m}{n}$. Q. E. D.

Corol. 1. Durationes duarum quarumlibet voluptatum æquabilium sunt in ratione composita ex ratione directâ quantitatum, & reciprocâ ratione Intensitatum.

Corol. 2. Intensitates duarum quarumlibet Voluptatum æquabilium sunt in ratione compositâ ex directâ quantitatum, & reciprocâ ratione durationum.

Corol. 3. Crescit Voluptas quælibet æquabilis in ratione durationum ab initio sumptarum.

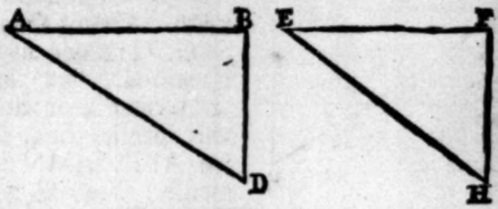
CAPUT IV.

De Voluptatibus uniformiter crescentibus.

Prop. XXVI. *Theor.* X.

QUantitates duarum Voluptatum uniformiter crescentium, quarum intensitates sub finem sunt æquales, sunt in ratione durationum.

Quoniam



Quoniam Figuræ quæ has voluptates repræsentant sunt triangula, (per *Def.* 5.) ideo sit unius quantitas v, duratio AB=r, intensitas sub finem BD=n; Alterius verò voluptatis quantitas sit V, duratio EF=s, & intensitas sub finem sit FH=m. Jam V. v :: EFH. ABD. sed EFH$=\frac{sm}{2}$, & ABD$=\frac{rn}{2}$: Ergo V. v :: $\frac{sm}{2}$. $\frac{rn}{2}$, unde $\frac{V}{v}=\frac{sm}{rn}$, sed m=n ex hypothesi, ergo $\frac{V}{v}=\frac{s}{r}$. Q. E. D.

Prop. XXVII. *Theor.* XI.

Quantitates duarum voluptatum uniformiter crescentium, quarum durationes sunt æquales, sunt in ratione directa Intensitatum sub finem sumptarum.

Nam designatis quantitatibus, ut in præcedenti, inventum fuit $\frac{V}{v}=\frac{sm}{rn}$, sed s=r ex hypothesi, Ergo $\frac{V}{v}=\frac{m}{n}$. Q. E. D.

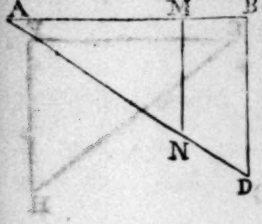
Prop. XXVIII. *Theor.* XII.

Quantitates duarum quarumlibet voluptatum uniformiter crescentium sunt in ratione compositâ ex ratione directâ durationum, & directâ ratione Intensitatum sub finem sumptarum.

Nam in propositione penultimâ demonstratum est $\frac{V}{v}=\frac{sm}{rn}$, id est, $\frac{V}{v}=\frac{s}{r}\times\frac{m}{n}$. Q. E. D.

Corol. 1. Durationes duarum voluptatum uniformiter crescentium, sunt in ratione compositâ ex directâ ratione quantitatum, & reciprocâ ratione intensitatum sub finem sumptarum.

Corol.



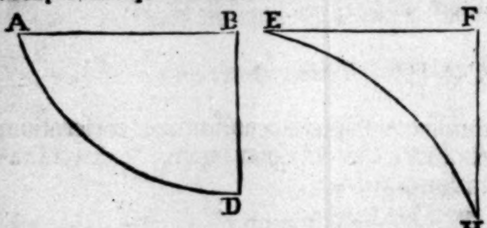
Corol. Quantitates unius voluptatis uniformiter crescentis ab initio sumptæ crescunt in duplicatâ ratione durationum. Sit $AB = T$ duratio una, ejusq; in hoc tempore quantitas sit Q; sit alia duratio $AM = t$, in qua voluptatis quantitas sit q. Jam Q. q :: ABD. AMN. sed ABD. AMN :: ABq. AMq (ex Elementis) Ergo Q. q :: ABq. AMq, id est Q. q :: T^2. t^2. Q. E. D.

CAPUT V.

De Voluptatibus, quarum Intensitates crescunt in ratione qualibet multiplicata aut submultiplicata.

Prop. XXIX. *Prob.* XVI.

DAtis æquationibus relationem exprimentibus inter tempora durationum & gradus intensitatum, invenire rationem quam habent voluptatum quantitates.



Sit unius voluptatis duratio $AB = r$, ejus intensitas sub finem $BD = n$, ejus quantitas v. Alterius autem quantitas sit V, duratio $EF = s$, & intensitas sub finem $FH = m$. Sitque $\overline{r}|^{c} = n$ æquatio exprimens relationem inter tempus durationis, & intensitatis gradus prioris crescentes juxta rationem quamlibet multiplicatam aut submultiplicatam, cujus exponens est c. Nec non $m = \overline{s}|^{e}$ æquatio exprimens relationem inter tempus durationis & intensitatis gradus posterioris crescentes in ratione quâlibet multiplicatâ aut submultiplicatâ, cujus exponens est e. Jam $\frac{V}{v} = \frac{EFH}{ABD}$ (juxta hujus methodi fundamentum sub initio Capitis tertii positum)

Sed

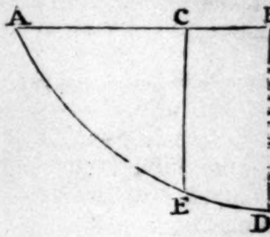
Sed per Quadraturarum Methodos notiſſimas $EFH = \frac{sm}{e+1}$, $ABD = \frac{rn}{c+1}$,

Ergo $\frac{V}{v} = \frac{sm \times \overline{c+1}}{rn \times \overline{e+1}} = \frac{s}{r} \times \frac{m}{n} \times \frac{c+1}{e+1}$. Id eſt, Voluptatum harum Quantitates ſunt in ratione compoſitâ ex directâ ratione temporum, directâ ratione intenſitatum (ſub finem ſumptarum) & reciprocâ ratione exponentium unitate auctarum. Q. E. I.

Scholium. Eſt hoc Theorema perquam generale; ejus enim ope inveniri poſſunt illa omnia, quæ ſpectant ad voluptates, quarum intenſitates creſcunt in quâvis ratione multiplicatâ aut ſubmultiplicatâ: ſic ſi ponatur $c=0$, $e=0$ habentur omnia in Capite III. demonſtrata; ſi ſit $c=1$, $e=1$, habentur omnia in Capite IV. demonſtrata; ſi ponas $c=0$, $e=1$, habetur ratio voluptatis æquabilis ad uniformiter creſcentem; vel ſi ponatur $c=1$, $e=2$, habetur ratio voluptatis uniformiter creſcentis ad voluptatem, cujus intenſitatis gradus creſcunt in duplicatâ ratione temporum: Ut de aliis infinitis nihil dicam, quæ pari facilitate ex hac propoſitione deduci poſſunt.

Prop. XXX. *Prob.* XVII.

Iiſdem datis quæ in præcedenti, invenire rationem incrementi unius voluptatis, pro diverſis ſuæ durationis temporibus ab initio ſumptis.



Sit Q quantitas voluptatis in tempore AB producta, & q quantitas ejuſdem (vel ſimilis) voluptatis in tempore AC producta. Ponatur $AB=T$, $AC=t$. Jam $BD=T^c$ & $CE=t^c$ ex ſuppoſito incremento graduum intenſitatis; Ergo per Quadraturas invenies $Q=\frac{T^{c+1}}{c+1} = ABD$, &

$q=\frac{t^{c+1}}{c+1} = ACE$. Unde $\frac{Q}{q} = \frac{T^{c+1}}{t^{c+1}}$. Q. E. I.

Corol. 1. Si voluptas ſit æquabilis, tum quantitates voluptatum erunt ut tempora, nam in hoc caſu $c=0$, unde $\frac{Q}{q} = \frac{T}{t}$.

Corol.

Corol. 2. Si Intenſitates creſcant uniformiter, voluptates erunt ut Quadrata temporum; nam in hoc caſu $c=1$, unde $\frac{Q}{q}=\frac{T^2}{t^2}$.

Corol. 3. Si Intenſitates creſcant in ratione temporum duplicatâ, voluptates erunt in ratione temporum triplicatâ; nam in hoc caſu $c=2$, unde $\frac{Q}{q}=\frac{T^3}{t^3}$.

Corol. 4. Si intenſitates creſcant in ſubduplicatâ ratione temporum, tum Quantitates voluptatis erunt ut Radix quadrata Cuborum Temporum; nam in hoc caſu $c=\frac{1}{2}$, unde $\frac{Q}{q}=\frac{\overline{T}|^{\frac{3}{2}}}{\overline{t}|^{\frac{3}{2}}}=\frac{\sqrt{T^3}}{\sqrt{t^3}}$.

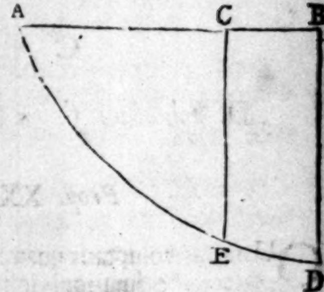
Corol. 5. Si intenſitates creſcant in ratione temporum ſubtriplicatâ, tum Quantitates voluptatis erunt ut Radix Cubica biquadratorum temporum. Nam in hoc caſu $c=\frac{1}{3}$, unde $\frac{Q}{q}=\frac{\overline{T}|^{\frac{4}{3}}}{\overline{t}|^{\frac{4}{3}}}=\frac{\sqrt[3]{T^4}}{\sqrt[3]{t^4}}$.

Scholium. Ex præmiſſis conſtat, quòd (conceſſis Figurarum Curvilinearum Quadraturis) poſſunt omnia ad voluptatum quantitates, mutuáſq; inter ſe relationes ſpectantia, facillimè inveniri. Quod ut melius intelligatur, adjiciam exemplum in ſequenti propoſitione, in quo intenſitates non creſcunt in ratione multiplicatâ aut ſubmultiplicatâ.

Prop. XXXI. *Prob.* XVIII.

Si intenſitas creſcat ut radix quadrata quadratorum & biquadratorum (ſimul ſumptorum) temporum; invenire rationem, juxta quam ipſa voluptas creſcat, pro diverſis durationis ſuæ temporibus, ab initio ſumptis.

Sit



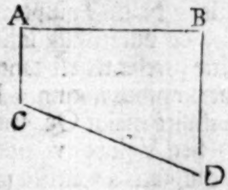
Sit AB=T una duratio, AC=t altera duratio; ſitq; BD intenſitas ſub finem prioris, CE intenſitas ſub finem alterius. Jam quia $BD=\sqrt{T^4+T^2}$, $CE=\sqrt{t^4+t^2}$ ex hypotheſi problematis, & per methodos Quadraturarum invenitur $ABD=\frac{1}{3}\times\overline{T^2+1}\sqrt{T^2+1}-1=Q$ & $ACE=\frac{1}{3}\times\overline{t^2+1}\sqrt{t^2+1}-1=q$. Ideo $\frac{Q}{q}=\frac{\overline{T^2+1}\sqrt{T^2+1}-1}{\overline{t^2+1}\sqrt{t^2+1}-1}$.

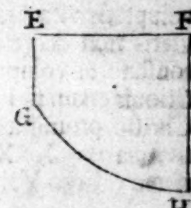
Exemplum ſit $T=3\frac{3}{7}$ horis, $t=2\frac{2}{5}$ horis, erit $\frac{Q}{q}=\frac{955125}{354662}$.

Scholium. Quamvis in præcedentibus ſuppoſui voluptates eſſe creſcentes, & intenſitates (exceptis voluptatibus æquabilibus) ſub initio eſſe indefinitè parvas; tamen eadem Methodus facilè applicari poſſit ad cæteras quaſvis voluptates à determinatâ intenſitatis magnitudine creſcentes aut decreſcentes: quarum in ſequenti propoſitione datur exemplum.

Prop. XXXII.

Sit unius voluptatis duratio AB, intenſitas ſub initio AC, ſub fine BD, quantitas v, & creſcant intenſitates ut Ordinatæ Trapezii triangularis ACBD; ſitq; alterius voluptatis duratio EB, intenſitas ſub initio EG, ſub fine FH, quantitas V, creſcantq; intenſitates ut ordinatæ trapezii parabolici EFGH; invenire rationem unius voluptatis ad alteram.





Ponantur AB=r, BD=n, AC=t, EF=s, FH=m, EG=g. Ex Geometriâ invenies $V=ACBD=\frac{nr+lr}{2}$

$V=\frac{2}{3}\times\overline{mg^2+ms-g^3}$;

Ergo $\frac{V}{v}=\frac{4mg^2+4ms-4g^3}{3nr+3lr}$.

Q. E. I.

CAPUT

CAPUT VI.

De Voluptate finitâ & infinitâ inter se comparatis.

Prop. XXXIII. *Theor.* XIII.

QUantitas voluptatis quamlibet habens (non decrescentem) intensitatem, & durationis infinitæ s, est infinitè major quâvis aliâ voluptate finitam semper habente intensitatem n, & durationis finitæ r.

Supponatur utraq; voluptas esse æquabilis, & V quantitas prioris, v posterioris. Jamq; erit $\frac{V}{v} = \frac{sm}{rn}$ (per *Prop.* XXV.) unde $V = \frac{sm}{rn} \times v$. Sed s est infinita, ergo productum sm est infinitum; & r, n sunt quantitates finitæ (ex hypothesi) Ergo productum rn est finitum; sed quantitas infinita sm, divisa per quantitatem finitam rn, dat quotientem infinitam $\frac{sm}{rn}$; Ergo $\frac{sm}{rn} \times v$ est Quantitas infinita; Ergo V quantitas voluptatis infinitæ durationis est infinitè major quantitate voluptatis v finitam habentis intensitatem & durationem. Q. E. D.

Scholium. Quamvis voluptates posui æquabiles, tamen præcedentia intelligenti facile constat Demonstrationem locum obtinere, sumptis intensitatibus in quavis datâ ratione crescentibus.

Corol. Valor voluptatis à Christo promissæ est infinitè major valore voluptatis vitæ præsentis. Nam Voluptas à Christo promissa est intensitatis non decrescentis, & durationis infinitæ, ut ex ipsius Historiâ constat: at voluptas vitæ præsentis est tantum intensitatis finitæ & durationis etiam finitæ, ut omnibus notum. Ergo Quantitas Voluptatis à Christo promissæ est infinitè major Quantitate Voluptatis Vitæ præsentis (per *Prop.* XXXIII.) Sed Valores voluptatum sunt in ratione Quantitatum; Ergo Valor Voluptatis à Christo promissæ, &c. Q. E. D.

Prop.

Prop. XXXIV. *Lemma* II.

Si probabilitas, quam habet aliquis ad obtinendum p, ſit ad probabilitatem, quam habet ad obtinendum P, in ratione quavis r ad 1, & ſupponatur P⊏⁻p, & r⊏⁻1, erit verus Valor expectationis illius hominis $\frac{P+p}{r+1}$: id eſt, ſumma rerum, quas expectat, diviſa per ſummam probabilitatum, dat verum expectationis ejus valorem.

Ut demonſtratio faciliùs capiatur; ſupponatur aliquis iſte A juſtum inire ludum cum altero homine B: ſitque x depoſitum ſeu valor expectationis, quam habet A; atque y depoſitum & valor expectationis, quam habet B (omnes enim Luſores in juſto ludo habent expectationem depoſitis ſuis æqualem) & ludant hac conditione, ut Victor det alteri p, ſervans ſibi ipſi P. Manifeſtum jam eſt, ſi A vincat, ipſum habiturum $x+y-p=P$; ſed ſi A perdat, tum (ex conditione Ludi) non niſi p habebit. Cúmque jam (ex hypotheſi Lemmatis) probabilitas, quam habet A ad vincendum (ſeu ut obtineat P) ſit ad probabilitatem, quam habet A ad perdendum (ſeu ut obtineat p) id eſt, ad probabilitatem, quam habet B ad vincendum, in ratione 1 ad r. Ideo, cùm juſtus ſupponitur Luſus, debent depoſita eſſe in ratione probabilitatum vincendi, id eſt, $x. y :: 1. r.$ unde $rx=y$; ſubſtituatur rx pro y in æquatione modò inventâ, & erit $x+rx=P+p$. unde $x=\frac{P+p}{r+1}$. Q. E. D.

Prop. XXXV. *Theor.* XIV.

Valor verus expectationis ad obtinendam voluptatem P à Chriſto promiſſam eſt infinitè major vero valore Expectationis obtinendi voluptatem p vitæ præſentis.

Nam P eſt infinitè major quàm p (per *Corol. Prop.* XXXIII.) & probabilitas obtinendi P eſt aliqua, eáq; non contemnenda (per *Prop.* XVII.) & probabilitas obtinendi p non eſt etiam niſi aliqua finita (ipſa enim vita, multóq; magìs vitæ hujus voluptas eſt incerta) ergo probabilitas obtinendi p eſt ad probabilitatem obtinendi P, ut numerus finitus ad numerum finitum; Ergo exprimi poſſit ratio harum probabilitatum, per rationem numeri finiti r ad unitatem. Ergo verus valor expectationis

...is est $\frac{P+p}{r+1}$ (per *Prop.* XXXIV.) Sed P est quantitas infinita (per ... *Prop.* XXXIII.) Ergo quantitas infinita $P+p$, divisa per nu... finitum scil. $r+1$, dat quotientem infinitam. Ergo verus va... expectationis Christiani est realiter infinitus; Ergo est infinitè ma... vero valore expectationis finito obtinendi vitæ hujus præsentis vo... Q. E. D.

Corol. 1. Conatus ad obtinendam vitæ futuræ voluptatem debent esse infinitè majores conatibus obtinendi vitæ hujus præsentis voluptatem; si sapienter conatus nostros gubernare vellemus (per hanc & *Axioma* 2.)

Corol. 2. Insipientes sunt, qui majorem adhibent conatum ad obtinendam præsentis, quàm vitæ futuræ voluptatem (per hanc & *Axioma* 3.)

Corol. 3. Minùs sapientes sunt, quorum conatus obtinendi voluptates ...tas sunt ad conatus obtinendi voluptates finitas in ratione finitâ (per hanc & *Axiomatis* part. 2.)

Corol. 4. Verus Christianus est omnium sapientûm sapientissimus, & Athei ac Deistæ sunt omnium stultorum stultissimi, sequitur ex *Corol.* ... & 2 hujus, & *Axiom.* 2 & 3.

FINIS.